Hoofdstuk 1: Vergelijkingen en ongelijkheden.

1.1 Tweedegraadsvergelijkingen

Opgave 1:
I, II, IV en V zijn tweedegraads vergelijkingen.
III is een eerstegraads vergelijking en VI is een derdegraads vergelijking.

Opgave 2:
a. $x^2 + 6 = 5x$
   $x^2 - 5x + 6 = 0$
   $(x - 3)(x - 2) = 0$
   $x = 3 \quad \vee \quad x = 2$

b. $x(x - 1) = 12$
   $x^2 - x - 12 = 0$
   $(x - 4)(x + 3) = 0$
   $x = 4 \quad \vee \quad x = -3$

c. $2x^2 = 5x$
   $2x^2 - 5x = 0$
   $x(2x - 5) = 0$
   $x = 0 \quad \vee \quad 2x = 5$
   $x = 0 \quad \vee \quad x = 2\frac{1}{2}$

d. $x = x^2$
   $-x^2 + x = 0$
   $-x(x - 1) = 0$
   $x = 0 \quad \vee \quad x = 1$

e. $x^2 = 11$
   $x = \sqrt{11} \quad \vee \quad x = -\sqrt{11}$

f. $x^2 + 4 = 1$
   $x^2 = -3$
   geen oplossingen

Opgave 3:
a. $3x^2 - 6x = 24$
   $3x^2 - 6x - 24 = 0$
   $x^2 - 2x - 8 = 0$
   $(x - 4)(x + 2) = 0$
   $x = 4 \quad \vee \quad x = -2$

b. $3x^2 - 6x = -3(x - 6)$
   $3x^2 - 6x = -3x + 18$
   $3x^2 - 3x - 18 = 0$
   $x^2 - x - 6 = 0$
   $(x - 3)(x + 2) = 0$
   $x = 3 \quad \vee \quad x = -2$

c. $2x^2 - 3x = 2$

$2x^2 - 3x - 2 = 0$

$x = \dfrac{3 \pm \sqrt{9 + 16}}{4} = \dfrac{3 \pm 5}{4}$

$x = \dfrac{3 + 5}{4} = 2 \quad \vee \quad x = \dfrac{3 - 5}{4} = -\tfrac{1}{2}$

d. $0{,}5x^2 - 2x - 6 = 0$

$x^2 - 4x - 12 = 0$

$(x - 6)(x + 2) = 0$

$x = 6 \quad \vee \quad x = -2$

e. $x^2 - 3x = 5(x - 3)$

$x^2 - 3x = 5x - 15$

$x^2 - 8x + 15 = 0$

$(x - 3)(x - 5) = 0$

$x = 3 \quad \vee \quad x = 5$

f. $2x^2 - 5x = 3x$

$2x^2 - 8x = 0$

$2x(x - 4) = 0$

$x = 0 \quad \vee \quad x = 4$

Opgave 4:

a. $6 - x^2 = -2$

$-x^2 = -8$

$x^2 = 8$

$x = \sqrt{8} \quad \vee \quad x = -\sqrt{8}$

b. $2x^2 = 9x + 5$

$2x^2 - 9x - 5 = 0$

$x = \dfrac{9 \pm \sqrt{81 + 40}}{4} = \dfrac{9 \pm 11}{4}$

$x = \dfrac{9 + 11}{4} = 5 \quad \vee \quad x = \dfrac{9 - 11}{4} = -0{,}5$

c. $3(x + 2)^2 + 5 = 80$

$3(x + 2)^2 = 75$

$(x + 2)^2 = 25$

$x + 2 = 5 \quad \vee \quad x + 2 = -5$

$x = 3 \quad \vee \quad x = -7$

d. $\tfrac{1}{2}(x - 3)^2 - 3 = 5$

$\tfrac{1}{2}(x - 3)^2 = 8$

$(x - 3)^2 = 16$

$x - 3 = 4 \quad \vee \quad x - 3 = -4$

$x = 7 \quad \vee \quad x = -1$

e. $-(2x-1)^2 + 5 = 1$

$-(2x-1)^2 = -4$

$(2x-1)^2 = 4$

$2x - 1 = 2 \quad \vee \quad 2x - 1 = -2$

$2x = 3 \quad \vee \quad 2x = -1$

$x = 1\frac{1}{2} \quad \vee \quad x = -\frac{1}{2}$

f. $8 - 3(4x-5)^2 = 5$

$-3(4x-5)^2 = -3$

$(4x-5)^2 = 1$

$4x - 5 = 1 \quad \vee \quad 4x - 5 = -1$

$4x = 6 \quad \vee \quad 4x = 4$

$x = 1\frac{1}{2} \quad \vee \quad x = 1$

Opgave 5:

a. $x^2 - 5x = 0$

$x(x-5) = 0$

$x = 0 \quad \vee \quad x = 5$

b. $x^2 - 5x = 14$

$x^2 - 5x - 14 = 0$

$(x-7)(x+2) = 0$

$x = 7 \quad \vee \quad x = -2$

c. $x^2 - 5 = 14$

$x^2 = 19$

$x = \sqrt{19} \quad \vee \quad x = -\sqrt{19}$

d. $x^2 - 5 = 14x$

$x^2 - 14x - 5 = 0$

$x = \dfrac{14 \pm \sqrt{196 + 20}}{2}$

$x = \dfrac{14 + \sqrt{216}}{2} \quad \vee \quad x = \dfrac{14 - \sqrt{216}}{2}$

e. $(2x-1)(3x+6) = 0$

$2x = 1 \quad \vee \quad 3x = -6$

$x = \frac{1}{2} \quad \vee \quad x = -2$

f. $(2x-1)(3x+6) = 9x$

$6x^2 + 12x - 3x - 6 = 9x$

$6x^2 = 6$

$x^2 = 1$

$x = 1 \quad \vee \quad x = -1$

g. $(2x-1) \cdot 3x = 6$

$6x^2 - 3x - 6 = 0$

$x = \dfrac{3 \pm \sqrt{9 + 144}}{12} = \dfrac{9 \pm \sqrt{153}}{12}$

h. $(2x-1)\cdot 3x = 6-9x$

$6x^2 - 3x + 9x - 6 = 0$

$6x^2 + 6x - 6 = 0$

$x^2 + x - 1 = 0$

$x = \dfrac{-1 \pm \sqrt{1+4}}{2} = \dfrac{-1 \pm \sqrt{5}}{2}$

Opgave 6:

a. $(x+3)^2 = 16x$

$x^2 + 6x + 9 = 16x$

$x^2 - 10x + 9 = 0$

$(x-9)(x-1) = 0$

$x = 9 \quad \vee \quad x = 1$

b. $(2x+3)^2 = -16$

geen oplossingen

c. $2(x+3)^2 = -4x$

$(x+3)^2 = -2x$

$x^2 + 6x + 9 = -2x$

$x^2 + 8x + 9 = 0$

$x = \dfrac{-8 \pm \sqrt{64-36}}{2} = \dfrac{-8 \pm \sqrt{28}}{2}$

d. $(2x+3)(4-x) = 9$

$8x - 2x^2 + 12 - 3x = 9$

$-2x^2 + 5x + 3 = 0$

$x = \dfrac{-5 \pm \sqrt{25+24}}{-4} = \dfrac{-5 \pm 7}{-4}$

$x = \dfrac{-5-7}{-4} = 3 \quad \vee \quad x = \dfrac{-5+7}{-4} = -\tfrac{1}{2}$

e. $(-4x+3)^2 = 36$

$-4x+3 = 6 \quad \vee \quad -4x+3 = -6$

$-4x = 3 \quad \vee \quad -4x = -9$

$x = -\tfrac{3}{4} \quad \vee \quad = 2\tfrac{1}{4}$

f. $-4(x+3)^2 = 4x$

$(x+3)^2 = -x$

$x^2 + 6x + 9 = -x$

$x^2 + 7x + 9 = 0$

$x = \dfrac{-7 \pm \sqrt{49-36}}{2} = \dfrac{-7 \pm \sqrt{13}}{2}$

g. $x^2 - (x+1)^2 = (x+3)^2$

$x^2 - (x^2 + 2x + 1) = x^2 + 6x + 9$

$x^2 - x^2 - 2x - 1 = x^2 + 6x + 9$

$-x^2 - 8x - 10 = 0$

$x^2 + 8x + 10 = 0$

$x = \dfrac{-8 \pm \sqrt{64 - 40}}{2} = \dfrac{-8 \pm \sqrt{24}}{2}$

h. $(x+3)^2 + (x+2)^2 = 25$

$x^2 + 6x + 9 + x^2 + 4x + 4 = 25$

$2x^2 + 10x - 12 = 0$

$x^2 + 5x - 6 = 0$

$(x+6)(x-1) = 0$

$x = -6 \quad \vee \quad x = 1$

Opgave 7:

a. $x^2 - x - 6 = 0$

$(x-3)(x+2) = 0$

$x = 3 \quad \vee \quad x = -2$

b. $x^2 + 2x - 6 = 0$

$D = 4 + 24 = 28 > 0$ dus twee oplossingen

c. $D = p^2 + 24 > 0$ klopt altijd want $p^2 \geq 0$ dus $p^2 + 24 \geq 24$

dus er zijn altijd twee oplossingen

Opgave 8:

a. $D = (-7)^2 - 4 \cdot 1 \cdot p = 49 - 4p > 0$

$-4p > -49$

$p < 12\tfrac{1}{4}$

b. $D = (-5)^2 - 4 \cdot 2 \cdot -p = 25 + 8p > 0$

$8p > -25$

$p > -3\tfrac{1}{8}$

c. $D = 4^2 - 4 \cdot -3 \cdot -p = 16 - 12p > 0$

$-12p > -16$

$p < 1\tfrac{1}{3}$

d. $D = (-3)^2 - 4 \cdot \tfrac{1}{4} \cdot p = 9 - p > 0$

$-p > -9$

$p < 9$

Opgave 9:

a. $D = p^2 - 4 \cdot 1 \cdot 25 = p^2 - 100 > 0$

$p^2 > 100$

$p < -10 \quad \vee \quad p > 10$

b. $D = p^2 - 4 \cdot 1 \cdot 4 = p^2 - 16 < 0$

$p^2 < 16$

$-4 < p < 4$

c. $D = p^2 - 4 \cdot -2 \cdot 3 = p^2 + 24 > 0$

dus voor iedere waarde van p zijn er twee oplossingen

Opgave 10:

a. $1 + 2 + p = 0$

$p = -3$

$x^2 + 2x - 3 = 0$

$(x + 3)(x - 1) = 0$

$x = -3 \quad \vee \quad x = 1$

b. $4p - 22 + 10 = 0$

$4p = 12$

$p = 3$

$3x^2 - 11x + 10 = 0$

$x = \dfrac{11 \pm \sqrt{121 - 120}}{6} = \dfrac{11 \pm 1}{6}$

$x = \dfrac{11 + 1}{6} = 2 \quad \vee \quad x = \dfrac{11 - 1}{6} = \tfrac{5}{3}$

Opgave 11:

a. $3x + 1 = 0$

$3x = -1$

$x = -\tfrac{1}{3}$ dus er is één oplossing

b. $D = 3^2 - 4 \cdot p \cdot 1 = 9 - 4p > 0 \quad \wedge \quad p \neq 0$

$-4p > -9$

$p < 2\tfrac{1}{4}$

dus $p < 0 \quad \vee \quad 0 < p < 2\tfrac{1}{4}$

Opgave 12:

a. $D = 5^2 - 4 \cdot p \cdot 2 = 25 - 8p > 0 \quad \wedge \quad p \neq 0$

$-8p > -25$

$p < 3\tfrac{1}{8}$

dus $p < 0 \quad \vee \quad 0 < p < 3\tfrac{1}{8}$

b. $D = (-3)^2 - 4 \cdot p \cdot -4 = 9 + 16p > 0 \quad \wedge \quad p \neq 0$

$16p > -9$

$p > -\tfrac{9}{16}$

dus $-\tfrac{9}{16} < p < 0 \quad \vee \quad p > 0$

Opgave 13:

a. $D = 1^2 - 4 \cdot 2 \cdot p = 1 - 8p < 0$

$-8p < -1$

$p > \frac{1}{8}$

b. $D = 1^2 - 4 \cdot p \cdot p = 1 - 4p^2 > 0 \quad \wedge \quad p \neq 0$

$-4p^2 > -1$

$p^2 < \frac{1}{4}$

$-\frac{1}{2} < p < \frac{1}{2}$

dus $-\frac{1}{2} < p < 0 \quad \vee \quad 0 < p < \frac{1}{2}$

c. $D = p^2 - 4 \cdot 2 \cdot 1 = p^2 - 8 > 0$

$p^2 > 8$

$p < -\sqrt{8} \quad \vee \quad p > \sqrt{8}$

Opgave 14:

a. $D = 6^2 - 4 \cdot p \cdot 9 = 36 - 36p = 0 \qquad \vee \qquad$ als $p = 0 \qquad 6x + 9 = 0$

$-36p = -36 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad 6x = -9$

$p = 1 \qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad x = -1\frac{1}{2}$

$x^2 + 6x + 9 = 0$

$(x + 3)(x + 3) = 0$

$x = -3$

b. $D = p^2 - 4 \cdot 1 \cdot 1 = p^2 - 4 = 0$

$p^2 = 4$

$p = 2 \quad \vee \quad p = -2$

$x^2 + 2x + 1 = 0 \qquad\qquad \vee \qquad x^2 - 2x + 1 = 0$

$(x + 1)(x + 1) = 0 \qquad\qquad\qquad (x - 1)(x - 1) = 0$

$x = -1 \qquad\qquad\qquad\qquad\qquad x = 1$

1.2 Hogeregraads- en modulusvergelijkingen

Opgave 15:
a. één oplossing, één oplossing
b. twee oplossingen, geen oplossingen

Opgave 16:
a.

x	x^2	x^3	x^4	x^5	x^6
1	1	1	1	1	1
2	4	8	16	32	64
3	9	27	81	243	729
4	16	64	256	1024	
5	25	125	625		
6	36	216			
7	49	343			
8	64				
9	81				

b. ***

Opgave 17:
a. $x^6 = 20$

$x = \sqrt[6]{20} \quad \vee \quad x = -\sqrt[6]{20}$

b. $5x^3 = 135$

$x^3 = 27$

$x = \sqrt[3]{27} = 3$

c. $0,5x^5 = 20$

$x^5 = 40$

$x = \sqrt[5]{40}$

d. $x^4 + 7 = 88$

$x^4 = 81$

$x = \sqrt[4]{81} = 3 \quad \vee \quad x = -3$

e. $1 - 3x^5 = 97$

$-3x^5 = 96$

$x^5 = -32$

$x = \sqrt[5]{-32} = -2$

f. $\frac{1}{4}x^8 + 3 = 10$

$\frac{1}{4}x^8 = 7$

$x^8 = 28$

$x = \sqrt[8]{28} \quad \vee \quad x = -\sqrt[8]{28}$

Opgave 18:

a. $5x^4 - 1 = 4$

$5x^4 = 5$

$x^4 = 1$

$x = 1 \quad \vee \quad x = -1$

b. $5x^4 = -4$

geen oplossingen

c. $5x^3 - 1 = 9$

$5x^3 = 10$

$x^3 = 2$

$x = \sqrt[3]{2}$

d. $8x^3 + 2 = 1$

$8x^3 = -1$

$x^3 = -\frac{1}{8}$

$x = \sqrt[3]{-\frac{1}{8}} = -\frac{1}{2}$

e. $5x^6 + 7 = 97$

$5x^6 = 90$

$x^6 = 18$

$x = \sqrt[6]{18} \quad \vee \quad x = -\sqrt[6]{18}$

f. $0,1x^7 - 1 = 999$

$0,1x^7 = 1000$

$x^7 = 10000$

$x = \sqrt[7]{10000}$

Opgave 19:

a. $3(x-2)^4 + 7 = 37$

$3(x-2)^4 = 30$

$(x-2)^4 = 10$

$x - 2 = \sqrt[4]{10} \quad \vee \quad x - 2 = -\sqrt[4]{10}$

$x = 2 + \sqrt[4]{10} \quad \vee \quad x = 2 - \sqrt[4]{10}$

b. $6 - (2x-1)^3 = 1$

$-(2x-1)^3 = -5$

$(2x-1)^3 = 5$

$2x - 1 = \sqrt[3]{5}$

$2x = 1 + \sqrt[3]{5}$

$x = \frac{1}{2} + \frac{1}{2} \cdot \sqrt[3]{5}$

c. $\frac{1}{2}(3x-1)^4 = 8$

$(3x-1)^4 = 16$

$3x - 1 = \sqrt[4]{16} = 2 \quad \vee \quad 3x - 1 = -2$

$3x = 3 \quad \vee \quad 3x = -1$

$$x = 1 \quad \lor \quad x = -\tfrac{1}{3}$$

d. $100 - \tfrac{1}{3}(4x-3)^5 = 19$

$-\tfrac{1}{3}(4x-3)^5 = -81$

$(4x-3)^5 = 243$

$4x - 3 = \sqrt[5]{243} = 3$

$4x = 6$

$x = 1\tfrac{1}{2}$

Opgave 20:

a. $5x^4 - 3 = 17$

$5x^4 = 20$

$x^4 = 4$

$x = \sqrt[4]{4} \quad \lor \quad x = -\sqrt[4]{4}$

b. $4x^3 - 5 = 1367$

$4x^3 = 1372$

$x^3 = 343$

$x = \sqrt[3]{343} = 7$

c. $3(4x-5)^3 = 15$

$(4x-5)^3 = 5$

$4x - 5 = \sqrt[3]{5}$

$4x = 5 + \sqrt[3]{5}$

$x = \tfrac{5}{4} + \tfrac{1}{4} \cdot \sqrt[3]{5}$

d. $17 - 2(1-3x)^4 = 5$

$-2(1-3x)^4 = -12$

$(1-3x)^4 = 6$

$1 - 3x = \sqrt[4]{6} \quad \lor \quad 1 - 3x = -\sqrt[4]{6}$

$-3x = -1 + \sqrt[4]{6} \quad \lor \quad -3x = -1 - \sqrt[4]{6}$

$x = \tfrac{1}{3} - \tfrac{1}{3} \cdot \sqrt[4]{6} \quad \lor \quad x = \tfrac{1}{3} + \tfrac{1}{3} \cdot \sqrt[4]{6}$

Opgave 21:

a. $x^3 - x^2 - 2x = 0$

$x(x^2 - x - 2) = 0$

$x(x-2)(x+1) = 0$

b. $x = 0 \quad \lor \quad x = 2 \quad \lor \quad x = -1$

Opgave 22:

a. $x^3 - 5x^2 + 6x = 0$

$x(x^2 - 5x + 6) = 0$

$x(x-2)(x-3) = 0$

$x = 0 \quad \lor \quad x = 2 \quad \lor \quad x = 3$

b. $x^3 - 5x^2 = 6x$

$x^3 - 5x^2 - 6x = 0$

$x(x^2 - 5x - 6) = 0$

$x(x - 6)(x + 1) = 0$

$x = 0 \quad \vee \quad x = 6 \quad \vee \quad x = -1$

c. $x^3 = 4x^2 + 12x$

$x^3 - 4x^2 - 12x = 0$

$x(x^2 - 4x - 12) = 0$

$x(x - 6)(x + 2) = 0$

$x = 0 \quad \vee \quad x = 6 \quad \vee \quad x = -2$

d. $x^4 - 13x^2 + 36 = 0$

stel $x^2 = p$

$p^2 - 13p + 36 = 0$

$(p - 4)(p - 9) = 0$

$p = 4 \quad \vee \quad p = 9$

$x^2 = 4 \quad \vee \quad x^2 = 9$

$x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = 3 \quad \vee \quad x = -3$

Opgave 23:

a. $x^4 - 10x^2 + 9 = 0$

stel $x^2 = p$

$p^2 - 10p + 9 = 0$

$(p - 1)(p - 9) = 0$

$p = 1 \quad \vee \quad p = 9$

$x^2 = 1 \quad \vee \quad x^2 = 9$

$x = 1 \quad \vee \quad x = -1 \quad \vee \quad x = 3 \quad \vee \quad x = -3$

b. $x^4 - 8x^2 - 9 = 0$

stel $x^2 = p$

$p^2 - 8p - 9 = 0$

$(p + 1)(p - 9) = 0$

$p = -1 \quad \vee \quad p = 9$

$x^2 = -1 \quad \vee \quad x^2 = 9$

k.n. $\qquad x = 3 \quad \vee \quad x = -3$

c. $x^4 + 16 = 10x^2$

$x^4 - 10x^2 + 16 = 0$

stel $x^2 = p$

$p^2 - 10p + 16 = 0$

$(p - 2)(p - 8) = 0$

$p = 2 \quad \vee \quad p = 8$

$x^2 = 2 \quad \vee \quad x^2 = 8$

$x = \sqrt{2} \quad \vee \quad x = -\sqrt{2} \quad \vee \quad x = \sqrt{8} \quad \vee \quad x = -\sqrt{8}$

d. $x^3 + 25x = 10x^2$

$x^3 - 10x^2 + 25x = 0$

$x(x^2 - 10x + 25) = 0$

$x(x - 5)(x - 5) = 0$

$x = 0 \quad \vee \quad x = 5$

Opgave 24:

a. $2x^4 - 11x^2 + 12 = 0$

stel $x^2 = p$

$2p^2 - 11p + 12 = 0$

$p = \dfrac{11 \pm \sqrt{121 - 96}}{4} = \dfrac{11 \pm \sqrt{25}}{4} = \dfrac{11 \pm 5}{4}$

$p = \dfrac{11 + 5}{4} = 4 \quad \vee \quad p = \dfrac{11 - 5}{4} = 1\tfrac{1}{2}$

b. $x^2 = 4 \quad \vee \quad x^2 = 1\tfrac{1}{2}$

$x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = \sqrt{1\tfrac{1}{2}} \quad \vee \quad x = -\sqrt{1\tfrac{1}{2}}$

Opgave 25:

a. $6x^4 + 2 = 7x^2$

$6x^4 - 7x^2 + 2 = 0$

stel $x^2 = p$

$6p^2 - 7p + 2 = 0$

$p = \dfrac{7 \pm \sqrt{49 - 48}}{12} = \dfrac{7 \pm 1}{12}$

$p = \dfrac{7 + 1}{12} = \tfrac{2}{3} \quad \vee \quad p = \dfrac{7 - 1}{12} = \tfrac{1}{2}$

$x^2 = \tfrac{2}{3} \quad \vee \quad x^2 = \tfrac{1}{2}$

$x = \sqrt{\tfrac{2}{3}} \quad \vee \quad x = -\sqrt{\tfrac{2}{3}} \quad \vee \quad x = \sqrt{\tfrac{1}{2}} \quad \vee \quad x = -\sqrt{\tfrac{1}{2}}$

b. $2x^4 = x^2 + 3$

$2x^4 - x^2 - 3 = 0$

stel $x^2 = p$

$2p^2 - p - 3 = 0$

$p = \dfrac{1 \pm \sqrt{1 + 24}}{4} = \dfrac{1 \pm 5}{4}$

$p = \dfrac{1 + 5}{4} = 1\tfrac{1}{2} \quad \vee \quad p = \dfrac{1 - 5}{4} = -1$

$x^2 = 1\tfrac{1}{2} \quad \vee \quad x^2 = -1$

$x = \sqrt{1\tfrac{1}{2}} \quad \vee \quad x = -\sqrt{1\tfrac{1}{2}}$

c. $4x^4 + 7x^2 = 2$

$4x^4 + 7x^2 - 2 = 0$

stel $x^2 = p$

$$4p^2 + 7p - 2 = 0$$

$$p = \frac{-7 \pm \sqrt{49 + 32}}{8} = \frac{-7 \pm 9}{8}$$

$$p = \frac{-7 + 9}{8} = \tfrac{1}{4} \quad \vee \quad p = \frac{-7 - 9}{8} = -2$$

$$x^2 = \tfrac{1}{4} \quad \vee \quad x^2 = -2$$

$$x = \tfrac{1}{2} \quad \vee \quad x = -\tfrac{1}{2}$$

d. $16x^4 + 225 = 136x^2$

$$16x^4 - 136x^2 + 225 = 0$$

stel $x^2 = p$

$$16p^2 - 136p + 225 = 0$$

$$p = \frac{136 \pm \sqrt{18496 - 14400}}{32} = \frac{136 \pm \sqrt{4096}}{32} = \frac{136 \pm 64}{32}$$

$$p = \frac{136 + 64}{32} = 6\tfrac{1}{4} \quad \vee \quad p = \frac{136 - 64}{32} = \tfrac{9}{4}$$

$$x^2 = 6\tfrac{1}{4} \quad \vee \quad x^2 = \tfrac{9}{4}$$

$$x = 2\tfrac{1}{2} \quad \vee \quad x = -2\tfrac{1}{2} \quad \vee \quad x = 1\tfrac{1}{2} \quad \vee \quad x = -1\tfrac{1}{2}$$

Opgave 26:

a. $4x^4 + 153 = 53x^2$

$$4x^4 - 53x^2 + 153 = 0$$

stel $x^2 = p$

$$4p^2 - 53p + 153 = 0$$

$$p = \frac{53 \pm \sqrt{2809 - 2448}}{8} = \frac{53 \pm \sqrt{361}}{8} = \frac{53 \pm 19}{8}$$

$$p = \frac{53 + 19}{8} = 9 \quad \vee \quad p = \frac{53 - 19}{8} = 4\tfrac{1}{4}$$

$$x^2 = 9 \quad \vee \quad x^2 = 4\tfrac{1}{4}$$

$$x = 3 \quad \vee \quad x = -3 \quad \vee \quad x = \sqrt{4\tfrac{1}{4}} \quad \vee \quad x = -\sqrt{4\tfrac{1}{4}}$$

b. $4x^4 + 21x^2 = 148$

$$4x^4 + 21x^2 - 148 = 0$$

stel $x^2 = p$

$$4p^2 + 21p - 148 = 0$$

$$p = \frac{-21 \pm \sqrt{441 + 2368}}{8} = \frac{-21 \pm \sqrt{2809}}{8} = \frac{-21 \pm 53}{8}$$

$$p = \frac{-21 + 53}{8} = 4 \quad \vee \quad p = \frac{-21 - 53}{8} = -9\tfrac{1}{4}$$

$$x^2 = 4 \quad \vee \quad x^2 = -9\tfrac{1}{4}$$

$$x = 2 \quad \vee \quad x = -2$$

c. $4x^6 + 35 = 24x^3$

$4x^6 - 24x^3 + 35 = 0$

stel $x^3 = p$

$4p^2 - 24p + 35 = 0$

$p = \dfrac{24 \pm \sqrt{576 - 560}}{8} = \dfrac{24 \pm \sqrt{16}}{8} = \dfrac{24 \pm 4}{8}$

$p = \dfrac{24 + 4}{8} = 3\tfrac{1}{2} \quad \vee \quad p = \dfrac{24 - 4}{8} = 2\tfrac{1}{2}$

$x^3 = 3\tfrac{1}{2} \quad \vee \quad x^3 = 2\tfrac{1}{2}$

$x = \sqrt[3]{3\tfrac{1}{2}} \quad \vee \quad x = \sqrt[3]{2\tfrac{1}{2}}$

d. $64x^6 + 27 = 224x^3$

$64x^6 - 224x^3 + 27 = 0$

stel $x^3 = p$

$64p^2 - 224p + 27 = 0$

$p = \dfrac{224 \pm \sqrt{50176 - 6912}}{128} = \dfrac{224 \pm \sqrt{43264}}{128} = \dfrac{224 \pm 208}{128}$

$p = \dfrac{224 + 208}{128} = 3\tfrac{3}{8} \quad \vee \quad p = \dfrac{224 - 208}{128} = \tfrac{1}{8}$

$x^3 = 3\tfrac{3}{8} \quad \vee \quad x^3 = \tfrac{1}{8}$

$x = \sqrt[3]{3\tfrac{3}{8}} = 1\tfrac{1}{2} \quad \vee \quad x = \sqrt[3]{\tfrac{1}{8}} = \tfrac{1}{2}$

Opgave 27:
a. -7 en 7
b. $2x - 1 = -7 \quad \vee \quad 2x - 1 = 7$

$2x = -6 \qquad \vee \quad 2x = 8$

$x = -3 \qquad \vee \quad x = 4$

Opgave 28:
a. $\left|2x - 1\right| = 8$

$2x - 1 = 8 \qquad \vee \quad 2x - 1 = -8$

$2x = 9 \qquad \vee \quad 2x = -7$

$x = 4\tfrac{1}{2} \qquad \vee \quad x = -3\tfrac{1}{2}$

b. $\left|x^2 - 3\right| = 1$

$x^2 - 3 = 1 \qquad \vee \quad x^2 - 3 = -1$

$x^2 = 4 \qquad \vee \quad x^2 = 2$

$x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = \sqrt{2} \quad \vee \quad x = -\sqrt{2}$

c. $\left|2x^2 - 5\right| = 11$

$2x^2 - 5 = 11 \qquad \vee \quad 2x^2 - 5 = -11$

$2x^2 = 16 \quad \vee \quad 2x^2 = -6$

$x^2 = 8 \qquad\qquad\qquad\quad$ k.n.

$x = \sqrt{8} \quad \vee \quad x = -\sqrt{8}$

d. $\left|5 - x^2\right| = 11$

$5 - x^2 = 11 \qquad \vee \quad 5 - x^2 = -11$

$-x^2 = 6 \qquad \vee \quad -x^2 = -16$

$x^2 = -6 \qquad \vee \quad x^2 = 16$

$\quad$ k.n. $\qquad\qquad x = 4 \quad \vee \quad x = -4$

Opgave 29:

a. $\left|2x^4 - 5\right| = 15$

$2x^4 - 5 = 15 \qquad \vee \quad 2x^4 - 5 = -15$

$2x^4 = 20 \;\vee\; 2x^4 = -10$

$x^4 = 10 \qquad\qquad\qquad$ k.n.

$x = \sqrt[4]{10} \quad \vee \quad x = -\sqrt[4]{10}$

b. $\left|2x^3 - 5\right| = 15$

$2x^3 - 5 = 15 \qquad \vee \quad 2x^3 - 5 = -15$

$2x^3 = 20 \;\vee\; 2x^3 = -10$

$x^3 = 10 \qquad\qquad \vee \quad x^3 = -5$

$x = \sqrt[3]{10} \quad \vee \quad x = \sqrt[3]{-5}$

c. $\left|x^4 - 5x^2\right| = 6$

$x^4 - 5x^2 = 6 \qquad\qquad\qquad \vee \quad x^4 - 5x^2 = -6$

$x^4 - 5x^2 - 6 = 0 \;\vee\; x^4 - 5x^2 + 6 = 0$

stel $x^2 = p$

$p^2 - 5p - 6 = 0 \;\vee\; p^2 - 5p + 6 = 0$

$(p - 6)(p + 1) = 0 \qquad \vee \quad (p - 2)(p - 3) = 0$

$p = 6 \quad \vee \quad p = -1 \quad \vee \quad p = 2 \quad \vee \quad p = 3$

$x^2 = 6 \quad \vee \quad x^2 = -1 \quad \vee \quad x^2 = 2 \quad \vee \quad x^2 = 3$

$x = \sqrt{6} \quad \vee \quad x = -\sqrt{6} \quad \vee \quad x = \sqrt{2} \quad \vee \quad x = -\sqrt{2} \quad \vee \quad x = \sqrt{3} \quad \vee \quad x = -\sqrt{3}$

d. $\left|x^6 - 10x^3\right| = 24$

$x^6 - 10x^3 = 24 \qquad\qquad \vee \quad x^6 - 10x^3 = -24$

$x^6 - 10x^3 - 24 = 0 \qquad \vee \quad x^6 - 10x^3 + 24 = 0$

stel $x^3 = p$

$p^2 - 10p - 24 = 0 \qquad\qquad \vee \quad p^2 - 10p + 24 = 0$

$(p - 12)(p + 2) = 0 \qquad \vee \quad (p - 4)(p - 6) = 0$

$p = 12 \quad \vee \quad p = -2 \quad \vee \quad p = 4 \quad \vee \quad p = 6$

$x^3 = 12 \quad \vee \quad x^3 = -2 \quad \vee \quad x^3 = 4 \quad \vee \quad x^3 = 6$

$x = \sqrt[3]{12} \quad \vee \quad x = \sqrt[3]{-2} \quad \vee \quad x = \sqrt[3]{4} \quad \vee \quad x = \sqrt[3]{6}$

1.3 Wortel- en gebroken vergelijkingen.

Opgave 30:

a. links en rechts kwadrateren

$$\sqrt{2x-5} = 3$$
$$2x - 5 = 9$$
$$2x = 14$$
$$x = 7$$

b. de uitkomst van een wortel kan nooit negatief zijn

Opgave 31:

a. $x = \sqrt{5x + 14}$

$$x^2 = 5x + 14$$
$$x^2 - 5x - 14 = 0$$
$$(x - 7)(x + 2) = 0$$
$$x = 7 \quad \vee \quad x = -2 \text{ (vervalt)}$$
dus $x = 7$

b. $3x = \sqrt{8x + 20}$

$$9x^2 = 8x + 20$$
$$9x^2 - 8x - 20 = 0$$
$$x = \frac{8 \pm \sqrt{64 + 720}}{18} = \frac{8 \pm \sqrt{784}}{18} = \frac{8 \pm 28}{18}$$
$$x = \frac{8 + 28}{18} = 2 \quad \vee \quad x = \frac{8 - 28}{18} = -1\tfrac{1}{9} \text{ (vervalt)}$$

c. $5\sqrt{x} = x$

$$25x = x^2$$
$$-x^2 + 25x = 0$$
$$-x(x - 25) = 0$$
$$x = 0 \quad \vee \quad x = 25$$

d. $3x = \sqrt{18x + 72}$

$$9x^2 = 18x + 72$$
$$9x^2 - 18x - 72 = 0$$
$$x^2 - 2x - 8 = 0$$
$$(x - 4)(x + 2) = 0$$
$$x = 4 \quad \vee \quad x = -2 \text{ (vervalt)}$$

Opgave 32:

a. $4 - 3\sqrt{x} = 2$

$$-3\sqrt{x} = -2$$
$$\sqrt{x} = \tfrac{2}{3}$$
$$x = \tfrac{4}{9}$$

b. $5\sqrt{x} - 2x = 0$

$$5\sqrt{x} = 2x$$

$$25x = 4x^2$$
$$-4x^2 + 25x = 0$$
$$-4x(x - 6\tfrac{1}{4}) = 0$$
$$x = 0 \quad \vee \quad x = 6\tfrac{1}{4}$$

c. $2x - 5\sqrt{x} = 3$
$$2x - 3 = 5\sqrt{x}$$
$$4x^2 - 12x + 9 = 25x$$
$$4x^2 - 37x + 9 = 0$$
$$x = \frac{37 \pm \sqrt{1369 - 144}}{8} = \frac{37 \pm \sqrt{1225}}{8} = \frac{37 \pm 35}{8}$$
$$x = 9 \quad \vee \quad x = \tfrac{1}{4} \text{ (vervalt)}$$

d. $5 - 2\sqrt{x} = 3$
$$-2\sqrt{x} = -2$$
$$\sqrt{x} = 1$$
$$x = 1$$

Opgave 33:

a. $2x + \sqrt{x} = 10$
$$2x - 10 = -\sqrt{x}$$
$$4x^2 - 40x + 100 = x$$
$$4x^2 - 41x + 100 = 0$$
$$x = \frac{41 \pm \sqrt{1681 - 1600}}{8} = \frac{41 \pm \sqrt{81}}{8} = \frac{41 \pm 9}{8}$$
$$x = \frac{41 + 9}{8} = 6\tfrac{1}{4} \text{ (vervalt)} \quad \vee \quad x = \frac{41 - 9}{8} = 4$$

b. $\sqrt{x + 12} = x$
$$x + 12 = x^2$$
$$-x^2 + x + 12 = 0$$
$$x^2 - x - 12 = 0$$
$$(x - 4)(x + 3) = 0$$
$$x = 4 \quad \vee \quad x = -3 \text{ (vervalt)}$$

c. $2x + \sqrt{x} = 6$
$$2x - 6 = -\sqrt{x}$$
$$4x^2 - 24x + 36 = x$$
$$4x^2 - 25x + 36 = 0$$
$$x = \frac{25 \pm \sqrt{625 - 576}}{8} = \frac{25 \pm \sqrt{49}}{8} = \frac{25 \pm 7}{8}$$
$$x = \frac{25 + 7}{8} = 4 \text{ (vervalt)} \quad \vee \quad x = \frac{25 - 7}{8} = 2\tfrac{1}{4}$$

d. $10 - x\sqrt{x} = 2$
$$-x\sqrt{x} = -8$$
$$x^3 = 64$$

$$x = \sqrt[3]{64} = 4$$

Opgave 34:

a. $p^2 + p - 6 = 0$

$(p + 3)(p - 2) = 0$

$p = -3 \quad \vee \quad p = 2$

b. $x\sqrt{x} = -3 \quad \vee \quad x\sqrt{x} = 2$

$\qquad$ k.n. $\qquad\qquad x^3 = 4$

$\qquad\qquad\qquad\qquad x = \sqrt[3]{4}$

Opgave 35:

a. $x^3 - 9x\sqrt{x} + 8 = 0$

stel $x\sqrt{x} = p$

$p^2 - 9p + 8 = 0$

$(p - 1)(p - 8) = 0$

$p = 1 \quad \vee \quad p = 8$

$x\sqrt{x} = 1 \qquad\quad \vee \quad x\sqrt{x} = 8$

$x^3 = 1 \qquad\qquad \vee \quad x^3 = 64$

$x = \sqrt[3]{1} = 1 \qquad \vee \quad x = \sqrt[3]{64} = 4$

b. $x^3 + 27 = 28x\sqrt{x}$

$x^3 - 28x\sqrt{x} + 27 = 0$

stel $x\sqrt{x} = p$

$p^2 - 28p + 27 = 0$

$(p - 1)(p - 27) = 0$

$p = 1 \quad \vee \quad p = 27$

$x\sqrt{x} = 1 \qquad\quad \vee \quad x\sqrt{x} = 27$

$x^3 = 1 \qquad\qquad \vee \quad x^3 = 729$

$x = \sqrt[3]{1} = 1 \qquad \vee \quad x = \sqrt[3]{729} = 9$

c. $8x^3 + 8 = 65x\sqrt{x}$

$8x^3 - 65x\sqrt{x} + 8 = 0$

stel $x\sqrt{x} = p$

$8p^2 - 65p + 8 = 0$

$$p = \frac{65 \pm \sqrt{4225 - 256}}{16} = \frac{65 \pm \sqrt{3969}}{16} = \frac{65 \pm 63}{16}$$

$$p = \frac{65 + 63}{16} = 8 \quad \vee \quad p = \frac{65 - 63}{16} = \tfrac{1}{8}$$

$x\sqrt{x} = 8 \qquad\qquad \vee \quad x\sqrt{x} = \tfrac{1}{8}$

$x^3 = 64 \qquad\qquad \vee \quad x^3 = \tfrac{1}{64}$

$x = \sqrt[3]{64} = 4 \qquad\quad \vee \quad x = \sqrt[3]{\tfrac{1}{64}} = \tfrac{1}{4}$

d. $x^5 - 33x^2\sqrt{x} + 32 = 0$

stel $x^2\sqrt{x} = p$

$p^2 - 33p + 32 = 0$

$(p-1)(p-32) = 0$

$p = 1 \qquad \vee \quad p = 32$

$x^2\sqrt{x} = 1 \vee \quad x^2\sqrt{x} = 32$

$x^5 = 1 \qquad \vee \quad x^5 = 1024$

$x = \sqrt[5]{1} = 1 \qquad \vee \quad x = \sqrt[5]{1024} = 4$

Opgave 36:

a. $x^3 + 30 = 11x\sqrt{x}$

$x^3 - 11x\sqrt{x} + 30 = 0$

stel $x\sqrt{x} = p$

$p^2 - 11p + 30 = 0$

$(p-5)(p-6) = 0$

$p = 5 \qquad \vee \quad p = 6$

$x\sqrt{x} = 5 \qquad \vee \quad x\sqrt{x} = 6$

$x^3 = 25 \qquad \vee \quad x^3 = 36$

$x = \sqrt[3]{25} \qquad \vee \quad x = \sqrt[3]{36}$

b. $x^3 + 125 = 126x\sqrt{x}$

$x^3 - 126x\sqrt{x} + 125 = 0$

stel $x\sqrt{x} = p$

$p^2 - 126p + 125 = 0$

$(p-1)(p-125) = 0$

$p = 1 \qquad \vee \quad p = 125$

$x\sqrt{x} = 1 \qquad \vee \quad x\sqrt{x} = 125$

$x^3 = 1 \qquad \vee \quad x^3 = 15625$

$x = \sqrt[3]{1} = 1 \qquad \vee \quad x = \sqrt[3]{15625} = 25$

c. $x^5 + 10 = 7x^2\sqrt{x}$

$x^5 - 7x^2\sqrt{x} + 10 = 0$

stel $x^2\sqrt{x} = p$

$p^2 - 7p + 10 = 0$

$(p-2)(p-5) = 0$

$p = 2 \qquad \vee \quad p = 5$

$x^2\sqrt{x} = 2 \qquad \vee \quad x^2\sqrt{x} = 5$

$x^5 = 4 \qquad \vee \quad x^5 = 25$

$x = \sqrt[5]{4} \qquad \vee \quad x = \sqrt[5]{25}$

d. $32x^5 + 32 = 1025x^2\sqrt{x}$

$32x^5 - 1025x^2\sqrt{x} + 32 = 0$

stel $x^2\sqrt{x} = p$

$32p^2 - 1025p + 32 = 0$

$$p = \frac{1025 \pm \sqrt{1050625 - 4096}}{64} = \frac{1025 \pm \sqrt{1046529}}{64} = \frac{1025 \pm 1023}{64}$$

$p = \dfrac{1025 + 1023}{64} = 32 \quad \vee \quad p = \dfrac{1025 - 1023}{64} = \frac{1}{32}$

$x^2\sqrt{x} = 32 \qquad\qquad \vee \quad x^2\sqrt{x} = \frac{1}{32}$

$x^5 = 1024 \qquad\qquad\quad \vee \quad x^5 = \frac{1}{1024}$

$x = \sqrt[5]{1024} = 4 \qquad \vee \quad x = \sqrt[5]{\frac{1}{1024}} = \frac{1}{4}$

Opgave 37:
Isoleren, kwadrateren en controleren.

Opgave 38:
a. kruiselings vermenigvuldigen

$x^2 = 2(x + 4)$

$x^2 = 2x + 8$

$x^2 - 2x - 8 = 0$

b. $(x - 4)(x + 2) = 0$

$x = 4 \quad \vee \quad x = -2$

Opgave 39:
a. $\dfrac{x + 3}{x - 1} = \dfrac{10}{x}$

$x(x + 3) = 10(x - 1)$

$x^2 + 3x = 10x - 10$

$x^2 - 7x + 10 = 0$

$(x - 2)(x - 5) = 0$

$x = 2 \quad \vee \quad x = 5$

b. $\dfrac{2x + 3}{x + 1} = \dfrac{2x + 2}{x - 1}$

$(2x + 3)(x - 1) = (2x + 2)(x + 1)$

$2x^2 - 2x + 3x - 3 = 2x^2 + 2x + 2x + 2$

$-3x = 5$

$x = -1\frac{2}{3}$

c. $\dfrac{x - 3}{x + 1} = 1\frac{1}{2}$

$\dfrac{x - 3}{x + 1} = \frac{3}{2}$

$2(x - 3) = 3(x + 1)$

$2x - 6 = 3x + 3$

$-x = 9$

$x = -9$

d. $\dfrac{x-1}{x}+1=3$

$\dfrac{x-1}{x}=2$

$x-1=2x$

$-x=1$

$x=-1$

e. $\dfrac{3x+4}{x-1}=\dfrac{x+18}{x}$

$x(3x+4)=(x+18)(x-1)$

$3x^2+4x=x^2-x+18x-18$

$2x^2-13x+18=0$

$x=\dfrac{13\pm\sqrt{169-144}}{4}=\dfrac{13\pm\sqrt{25}}{4}=\dfrac{13\pm5}{4}$

$x=\dfrac{13+5}{4}=4\tfrac{1}{2}\quad\vee\quad x=\dfrac{13-5}{4}=2$

f. $\dfrac{2x-5}{4-x}=\dfrac{x+2}{3x-4}$

$(2x-5)(3x-4)=(x+2)(4-x)$

$6x^2-8x-15x+20=4x-x^2+8-2x$

$7x^2-25x+12=0$

$x=\dfrac{25\pm\sqrt{625-336}}{14}=\dfrac{25\pm\sqrt{289}}{14}=\dfrac{25\pm17}{14}$

$x=\dfrac{25+17}{14}=3\quad\vee\quad x=\dfrac{25-17}{14}=\tfrac{4}{7}$

Opgave 40:

a. $\dfrac{5x^2-15}{x^2+5}=0$

$5x^2-15=0$

$5x^2=15$

$x^2=3$

$x=\sqrt{3}\quad\vee\quad x=-\sqrt{3}$

b. $\dfrac{x^2-3}{x^2+1}=\dfrac{x-1}{x^2+1}$

$x^2-3=x-1$

$x^2-x-2=0$

$(x-2)(x+1)=0$

$x=2\quad\vee\quad x=-1$

c. $\dfrac{x^2-4}{2x+5}=\dfrac{x^2-4}{x+4}$

$x^2-4=0\qquad\qquad\vee\quad 2x+5=x+4$

$x^2=4\qquad\qquad\quad\vee\quad x=-1$

$$x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = -1$$

d. $\dfrac{x^2 + 1}{x + 1} = \dfrac{x + 3}{x + 1}$

$x^2 + 1 = x + 3$

$x^2 - x - 2 = 0$

$(x - 2)(x + 1) = 0$

$x = 2 \quad \vee \quad x = -1$ (vervalt)

Opgave 41:

a. $\dfrac{3x^2 - 10}{x^2 + 1} = 2$

$3x^2 - 10 = 2(x^2 + 1)$

$3x^2 - 10 = 2x^2 + 2$

$x^2 = 12$

$x = \sqrt{12} \quad \vee \quad x = -\sqrt{12}$

b. $\dfrac{x^3 - 8}{x^2 + 2} = \dfrac{x^3 - 8}{x + 8}$

$x^3 = 8 \quad \vee \quad x^2 + 2 = x + 8$

$x = \sqrt[3]{8} = 2 \quad \vee \quad x^2 - x - 6 = 0$

$x = 2 \quad \vee \quad (x - 3)(x + 2) = 0$

$x = 2 \quad \vee \quad x = 3 \quad \vee \quad x = -2$

c. $\dfrac{3x^2 - 10}{(x^2 + 1)^2} = \tfrac{2}{25}$

$25(3x^2 - 10) = 2(x^2 + 1)^2$

$75x^2 - 250 = 2(x^4 + 2x^2 + 1)$

$75x^2 - 250 = 2x^4 + 4x^2 + 2$

$-2x^4 + 71x^2 - 252 = 0$

stel $x^2 = p$

$-2p^2 + 71p - 252 = 0$

$p = \dfrac{-71 \pm \sqrt{5041 - 2016}}{-4} = \dfrac{-71 \pm \sqrt{3025}}{-4} = \dfrac{-71 \pm 55}{-4}$

$p = \dfrac{-71 + 55}{-4} = 4 \qquad \vee \qquad p = \dfrac{-71 - 55}{-4} = 31\tfrac{1}{2}$

$x^2 = 4 \quad \vee \quad x^2 = 31\tfrac{1}{2}$

$x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = \sqrt{31\tfrac{1}{2}} \quad \vee \quad x = -\sqrt{31\tfrac{1}{2}}$

d. $\dfrac{6x^2 - 12}{(x^2 - 1)^2} = 1\tfrac{1}{3}$

$\dfrac{6x^2 - 12}{(x^2 - 1)^2} = \tfrac{4}{3}$

$3(6x^2 - 12) = 4(x^2 - 1)^2$

$18x^2 - 36 = 4(x^4 - 2x^2 + 1)$

$$18x^2 - 36 = 4x^4 - 8x^2 + 4$$
$$-4x^4 + 26x^2 - 40 = 0$$
stel $x^2 = p$
$$-4p^2 + 26p - 40 = 0$$
$$p = \frac{-26 \pm \sqrt{676 - 640}}{-8} = \frac{-26 \pm \sqrt{36}}{-8} = \frac{-26 \pm 6}{-8}$$
$$p = \frac{-26 + 6}{-8} = 2\tfrac{1}{2} \quad \vee \quad p = \frac{-26 - 6}{-8} = 4$$
$$x^2 = 2\tfrac{1}{2} \quad \vee \quad x^2 = 4$$
$$x = \sqrt{2\tfrac{1}{2}} \quad \vee \quad x = -\sqrt{2\tfrac{1}{2}} \quad \vee \quad x = 2 \quad \vee \quad x = -2$$

1.4 Stelsels vergelijkingen

Opgave 42:

a. ja, $0 + 4 \cdot 3 = 12$

b. ja, $-\frac{1}{4} \cdot 4 + 3 = 2$

c. $y = -\frac{1}{4}x + 3$

$\frac{1}{4}x + y = 3$

$x + 4y = 12$

Opgave 43:

$l: \quad 3x - y = 6$

x	0	2
y	-6	0

$m: \quad x + y = 1$

x	0	1
y	1	0

$n: \quad x - y = 0$

x	0	1
y	0	1

$p: \quad x + 2y = 4$

x	0	4
y	2	0
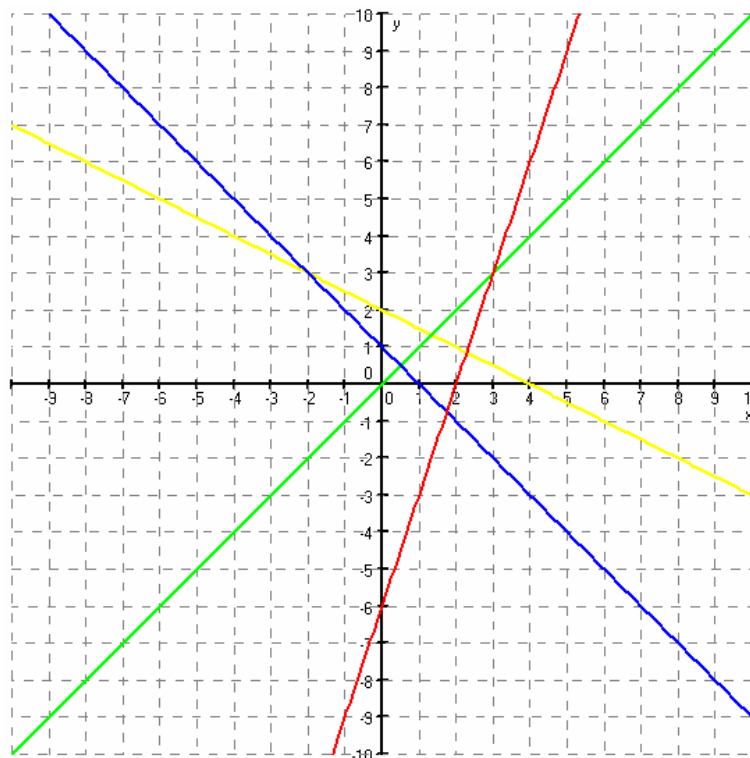
Opgave 44:

a. x-as: $y = 0$

$4x = 24$

$x = 6$ dus $(6,0)$

y-as: $x = 0$

$-3y = 24$

$y = -8$ dus $(0,-8)$

b. A: $4 \cdot 8 - 3 \cdot 3 = 23 \neq 24$ dus A ligt niet op l

B: $4 \cdot 18 - 3 \cdot 16 = 24$ dus B ligt wel op l

C: $4 \cdot -30 - 3 \cdot -48 = 24$ dus C ligt wel op l

c. $4 \cdot 16 - 3p = 24$

$64 - 3p = 24$

$-3p = -40$

$p = 13\frac{1}{3}$

d. $4q - 3 \cdot 48 = 24$
$4q - 144 = 24$
$4q = 168$
$q = 42$

Opgave 45:

a. $l: \quad 2x + y = 3$

x	0	$1\frac{1}{2}$
y	3	0

$m: \quad x - 2y = 4$

x	0	4
y	-2	0

b. $(2,-1)$

c. $x = 2 \quad \wedge \quad y = -1$
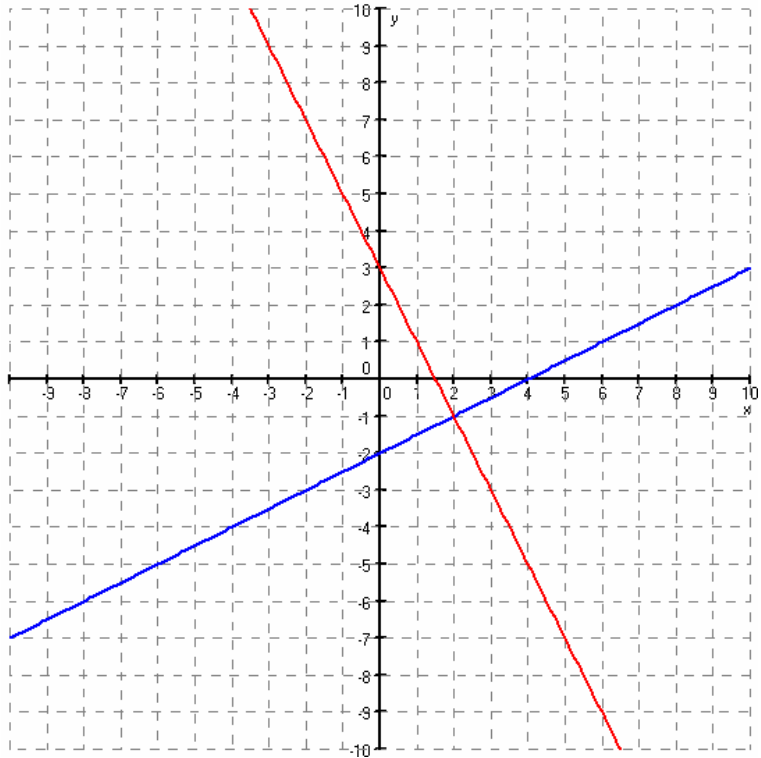
Opgave 46:

a. $\begin{cases} 5x - 4y = -8 \\ -x + 4y = -12 \end{cases} +$
$\overline{\qquad 4x = -20}$

$x = -5$
$-25 - 4y = -8$
$-4y = 17$
$y = -4\frac{1}{4}$
$x = -5 \quad \wedge \quad y = -4\frac{1}{4}$

b. $\begin{cases} -2x + y = 7 \\ -2x + 3y = -1 \end{cases} -$
$\overline{\qquad -2y = 8}$

$y = -4$
$-2x - 4 = 7$
$-2x = 11$
$x = -5\frac{1}{2}$
$x = -5\frac{1}{2} \quad \wedge \quad y = -4$

c.
$$\begin{cases} -x - 3y = -8 \\ -2x + 3y = -1 \quad + \end{cases}$$
$$-3x = -9$$
$$x = 3$$
$$-3 - 3y = -8$$
$$-3y = -5$$
$$y = \tfrac{5}{3} = 1\tfrac{2}{3}$$
$$x = 3 \quad \wedge \quad y = 1\tfrac{2}{3}$$

Opgave 47:

a.
$$\begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \quad + \end{cases}$$
$$5x - y = 23$$
nee

b.
$$\begin{cases} 3x - 4y = 7 \\ 2x + 3y = 16 \quad - \end{cases}$$
$$x - 7y = -9$$
nee

Opgave 48:

a.
$$\begin{cases} 3x + 5y = -7 & |\times 1 \\ 2x + y = 0 & |\times 5 \end{cases}$$
$$\begin{cases} 3x + 5y = -7 \\ 10x + 5y = 0 \quad - \end{cases}$$
$$-7x = -7$$
$$x = 1$$
$$3 + 5y = -7$$
$$5y = -10$$
$$y = -2$$
$$x = 1 \quad \wedge \quad y = -2$$

b.
$$\begin{cases} 2x - 4y = 6 & |\times 1 \\ 3x - y = 19 & |\times 4 \end{cases}$$
$$\begin{cases} 2x - 4y = 6 \\ 12x - 4y = 76 \quad - \end{cases}$$
$$-10x = -70$$
$$x = 7$$
$$14 - 4y = 6$$
$$-4y = -8$$
$$y = 2$$
$$x = 7 \quad \wedge \quad y = 2$$

c. $\begin{cases} 4x + y = 13 & \Big| \times 2 \\ x - 2y = 1 & \Big| \times 1 \end{cases}$

$\begin{cases} 8x + 2y = 26 \\ x - 2y = 1 \quad + \end{cases}$
$\overline{\qquad 9x = 27}$

$x = 3$
$12 + y = 13$
$y = 1$
$x = 3 \quad \wedge \quad y = 1$

Opgave 49:

a. $\begin{cases} 5x + 2y = 69 & \Big| \times 1 \\ x + 3y = -7 & \Big| \times 5 \end{cases}$

$\begin{cases} 5x + 2y = 69 \\ 5x + 15y = -35 \quad - \end{cases}$
$\overline{\qquad -13y = 104}$

$y = -8$
$x - 24 = -7$
$x = 17$
$x = 17 \quad \wedge \quad y = -8$

b. $\begin{cases} 2x - 5y = -19 & \Big| \times 4 \\ 5x + 4y = 35 & \Big| \times 5 \end{cases}$

$\begin{cases} 8x - 20y = -78 \\ 25x + 20y = 175 \quad + \end{cases}$
$\overline{\qquad 33x = 99}$

$x = 3$
$15 + 4y = 35$
$4y = 20$
$y = 5$
$x = 3 \quad \wedge \quad y = 5$

c. $\begin{cases} 0{,}8x + 0{,}2y = 1 & \Big| \times 3 \\ 0{,}3x - 0{,}3y = 1{,}5 & \Big| \times 2 \end{cases}$

$\begin{cases} 2{,}4x + 0{,}6y = 3 \\ 0{,}6x - 0{,}6y = 3 \quad + \end{cases}$
$\overline{\qquad 3x = 6}$

$x = 2$
$1{,}6 + 0{,}2y = 1$
$0{,}2y = -0{,}6$
$y = -3$
$x = 2 \quad \wedge \quad y = -3$

Opgave 50:

$$\begin{cases} 3x - 2y = -12 \\ x + 4y = 38 \end{cases} \quad \begin{matrix} \times 2 \\ \times 1 \end{matrix}$$

$$\begin{cases} 6x - 4y = -24 \\ x + 4y = 38 \quad + \end{cases}$$
$$7x = 14$$

$x = 2$
$6 - 2y = -12$
$-2y = -18$
$y = 9$
$S = (2,9)$

Opgave 51:

a. $1^2 + b + c = -2$
   $b + c = -3$

b. $2^2 + 2b + c = 3$
   $2b + c = -1$

c. $\begin{cases} b + c = -3 \\ 2b + c = -1 \quad - \end{cases}$
   $-b = -2$

   $b = 2$
   $2 + c = -3$
   $c = -5$
   $b = 2 \quad \wedge \quad c = -5$

Opgave 52:

$$\begin{cases} a + c = 8 \\ 4a + c = 17 \quad - \end{cases}$$
$$-3a = -9$$

$a = 3$
$3 + c = 8$
$c = 5$
$a = 3 \quad \wedge \quad c = 5$
$y = 3x^2 + 5$

Opgave 53:

$$\begin{cases} 2a + b = 8 \\ 2b + a = 8 \end{cases} \quad \begin{matrix} \times 2 \\ \times 1 \end{matrix}$$

$$\begin{cases} 4a + 2b = 16 \\ a + 2b = 8 \quad - \end{cases}$$
$$3a = 8$$

$a = 2\frac{2}{3}$
$2\frac{2}{3} + 2b = 8$

$2b = 5\frac{1}{3}$

$b = 2\frac{2}{3}$

$a = 2\frac{2}{3} \quad \wedge \quad b = 2\frac{2}{3}$

Opgave 54:

a. $\begin{cases} 4 + 2p + q = -1 \\ \quad 4p - q = -1 \quad + \end{cases}$

$\qquad \overline{\quad 4 + 6p = -2 \quad}$

$6p = -6$

$p = -1$

$4 - 2 + q = -1$

$q = -3$

$p = -1 \quad \wedge \quad q = -3$

b. $y = x^2 - x - 3$ en $y = -2x + 3$

$x^2 - x - 3 = -2x + 3$

$x^2 + x - 6 = 0$

$(x + 3)(x - 2) = 0$

$x = -3 \quad \vee \quad x = 2$

$y = 9 \quad \vee \quad y = -1$

dus $(-3,9)$ en $(2,-1)$

Opgave 55:

Door $(0,4)$ dus $c = 4$

$y = ax^2 + bx + 4$

$\begin{cases} 4a - 2b + 4 = -10 \\ 9a + 3b + 4 = 5 \end{cases}$

$\begin{cases} 4a - 2b = -14 & |\times 3 \\ 9a + 3b = 1 & |\times 2 \end{cases}$

$\begin{cases} 12a - 6b = -42 \\ 18a + 6b = 2 \qquad + \end{cases}$

$\qquad \overline{\quad 30a = -40 \quad}$

$a = -1\frac{1}{3}$

$-5\frac{1}{3} - 2b + 4 = -10$

$-2b = -8\frac{2}{3}$

$b = 4\frac{1}{3}$

$y = -1\frac{1}{3}x^2 + 4\frac{1}{3}x + 4$

Opgave 56:

$\begin{cases} 2x + 3y = 12 \\ \quad y = 4x - 10 \end{cases}$

$2x + 3(4x - 10) = 12$

$2x + 12x - 30 = 12$
$14x = 42$
$x = 3$
$y = 2$
dus $S = (3,2)$

Opgave 57:
a. $2x + 2y = 9 \quad \wedge \quad y = 4x - 3$
    $2x + 2(4x - 3) = 9$
    $2x + 8x - 6 = 9$
    $10x = 15$
    $x = 1\frac{1}{2}$
    $y = 3$
    dus $x = 1\frac{1}{2} \quad \wedge \quad y = 3$
b. $y = \frac{1}{2}x + 1 \quad \wedge \quad 3x + 6y = 8$
    $3x + 6(\frac{1}{2}x + 1) = 8$
    $3x + 3x + 6 = 8$
    $6x = 2$
    $x = \frac{1}{3}$
    $y = 1\frac{1}{6}$
    dus $x = \frac{1}{3} \quad \wedge \quad y = 1\frac{1}{6}$
c. $x = 5y - 3 \quad \wedge \quad 3x + 4y = 29$
    $3(5y - 3) + 4y = 29$
    $15y - 9 + 4y = 29$
    $19y = 38$
    $y = 2$
    $x = 7$
    dus $x = 7 \quad \wedge \quad y = 2$

Opgave 58:
a. $\begin{cases} y = x^2 - 3 \\ x - y = -3 \end{cases}$
    $x - (x^2 - 3) = -3$
    $x - x^2 + 3 = -3$
    $-x^2 + x + 6 = 0$
    $x^2 - x - 6 = 0$
    $(x - 3)(x + 2) = 0$
    $x = 3 \quad \vee \quad x = -2$
    $y = 6 \quad \vee \quad y = 1$
    dus $(x = 3 \quad \wedge \quad y = 6) \quad \vee \quad (x = -2 \quad \wedge \quad y = 1)$

b. $\begin{cases} x^2 + y^2 = 25 \\ 3x + y = 5 \end{cases}$

$\begin{cases} x^2 + y^2 = 25 \\ y = -3x + 5 \end{cases}$

$x^2 + (-3x + 5)^2 = 25$

$x^2 + 9x^2 - 30x + 25 = 25$

$10x^2 - 30x = 0$

$10x(x - 3) = 0$

$x = 0 \quad \vee \quad x = 3$

$y = 5 \quad \vee \quad y = -4$

dus $(x = 0 \quad \wedge \quad y = 5) \quad \vee \quad (x = 3 \quad \wedge \quad y = -4)$

c. $\begin{cases} x^2 + y^2 = 20 \\ xy = 8 \end{cases}$

$\begin{cases} x^2 + y^2 = 20 \\ y = \dfrac{8}{x} \end{cases}$

$x^2 + \left(\dfrac{8}{x}\right)^2 = 20$

$x^2 + \dfrac{64}{x^2} = 20$

$x^4 + 64 = 20x^2$

$x^4 - 20x^2 + 64 = 0$

stel $x^2 = p$

$p^2 - 20p + 64 = 0$

$(p - 4)(p - 16) = 0$

$p = 4 \quad \vee \quad p = 16$

$x^2 = 4 \quad \vee \quad x^2 = 16$

$x = 2 \quad \vee \quad x = -2 \quad \vee \quad x = 4 \quad \vee \quad x = -4$

$y = 4 \quad \vee \quad y = -4 \quad \vee \quad y = 2 \quad \vee \quad y = -2$

dus $(x = 2 \quad \wedge \quad y = 4) \quad \vee \quad (x = -2 \quad \wedge \quad y = -4) \quad \vee \quad (x = 4 \quad \wedge \quad y = 2) \quad \vee$

$(x = -4 \quad \wedge \quad y = -2)$

1.5 Grafisch-numeriek oplossen

Opgave 59:

I: $x^4 - x^2 - 2 = 0$

     stel $x^2 = p$

     $p^2 - p - 2 = 0$

     $(p - 2)(p + 1) = 0$

     $p = 2 \quad \vee \quad p = -1$

     $x^2 = 2 \quad \vee \quad x^2 = -1$

     $x = \sqrt{2} \quad \vee \quad x = -\sqrt{2}$

II: $x^4 - x^3 - 2 = 0$ niet

III: $x^4 - x^3 - 2x = 0$

     $x(x^3 - x^2 - 2) = 0$

     $x = 0 \quad \vee \quad x^3 - x^2 - 2 = 0$ niet

IV: $x^4 - x^3 - 2x^2 = 0$

     $x^2(x^2 - x - 2) = 0$

     $x^2(x - 2)(x + 1) = 0$

     $x = 0 \quad \vee \quad x = 2 \quad \vee \quad x = -1$

Opgave 60:

a. $x = -1 \quad \vee \quad x = 1 \quad \vee \quad x = 2 \quad \vee \quad x = 3$

b. $(-1)^4 - 5 \cdot (-1)^3 + 5 \cdot (-1)^2 + 5 \cdot -1 - 6 = 1 + 5 + 5 - 5 - 6 = 0$

    $1^4 - 5 \cdot 1^3 + 5 \cdot 1^2 + 5 \cdot -1 - 6 = 1 - 5 + 5 + 5 - 6 = 0$

    $2^4 - 5 \cdot 2^3 + 5 \cdot 2^2 + 5 \cdot 2 - 6 = 16 - 40 + 20 + 10 - 6 = 0$

    $3^4 - 5 \cdot 3^3 + 5 \cdot 3^2 + 5 \cdot 3 - 6 = 81 - 135 + 45 + 15 - 6 = 0$

Opgave 61:

a. $x = -2 \quad \vee \quad x = 2 \quad \vee \quad x = 4$

b. $x = -2 \quad \vee \quad x = 2 \quad \vee \quad x = 4$

Opgave 62:

a. $y_1 = x^3 - 4x^2 + 3$ met de optie zero: $x = -0{,}79 \quad \vee \quad x = 1 \quad \vee \quad x = 3{,}79$

b. $y_1 = x^4 - 4x^3 + 2x^2 + x - 1 = 0$ met de optie zero: $x = -0{,}58 \quad \vee \quad x = 3{,}34$

c. $y_1 = 0{,}4x^3 + 2x^2 + x - 2$ en $y_2 = x + 2$ met de optie intersect:

    $x = -4{,}51 \quad \vee \quad -1{,}76 \quad \vee \quad x = 1{,}26$

d. $y_1 = 0{,}2x^5 - x^4 + 4x^2$ en $y_2 = 0{,}2x + 3$ met de optie intersect:

    $x = -1{,}45 \quad \vee \quad x = -1 \quad \vee \quad x = 1 \quad \vee \quad x = 3 \quad \vee \quad x = 3{,}45$

Opgave 63:

a. $y_1 = 0{,}5x^3 - 5x^2 + 20$ met de optie zero: $x = -1{,}84 \quad \vee \quad x = 2{,}28 \quad \vee \quad x = 9{,}56$

b. $y_1 = 0{,}1x^4 + 0{,}1x^3 - 12x^2 + 50$ en $y_2 = 25x$ met de optie intersect:

    $x = -10 \quad \vee \quad x = -3{,}53 \quad \vee \quad x = 1{,}26 \quad \vee \quad x = 11{,}27$

Opgave 64:

a. $y_1 = abs(x^3 - 9x)$ en $y_2 = 5$ met de optie intersect:

$x = -3,25$ $\lor$ $x = -2,67$ $\lor$ $x = -0,58$ $\lor$ $x = 0,58$ $\lor$ $x = 2,67$ $\lor$ $x = 3,25$

b. $y_1 = abs(x^3 - 9x)$ en $y_2 = x + 5$ met de optie intersect:

$x = -3,10$ $\lor$ $x = -2,87$ $\lor$ $x = -0,51$ $\lor$ $x = 0,66$ $\lor$ $x = 2,44$ $\lor$ $x = 3,39$

Opgave 65:

a. $y_1 = abs(x^4 - x^3 + x - 5)$ en $y_2 = x + 3$ met de optie intersect:

$x = -1,48$ $\lor$ $x = -1,26$ $\lor$ $x = 1$ $\lor$ $x = 2$

b. $y_1 = abs(x^3 - 5x^2 - 2x + 24)$ en $y_2 = 20$ met de optie intersect:

$x = -2,55$ $\lor$ $x = -1$ $\lor$ $x = 0,76$ $\lor$ $x = 5,24$

c. $y_1 = abs(x^2 - 4x)$ en $y_2 = abs(x^2 + 2x - 3)$ met de optie intersect:

$x = -0,82$ $\lor$ $x = 0,50$ $\lor$ $x = 1,82$

d. $y_1 = abs(x^3 - 4x^2 - 3x + 10)$ met de optie zero:

$x = 1,48$ $\lor$ $x = 4,14$

Opgave 66:

a. $y_1 = -x^2 + 6x$ en $y_2 = x + 4$ met de optie intersect: $x = 1$ $\lor$ $x = 4$

b. $1 < x < 4$

Opgave 67:

a. $y_1 = x^2 - 3x$ en $y_2 = 14$
met de optie intersect:
$x = -2,53$ $\lor$ $x = 5,53$
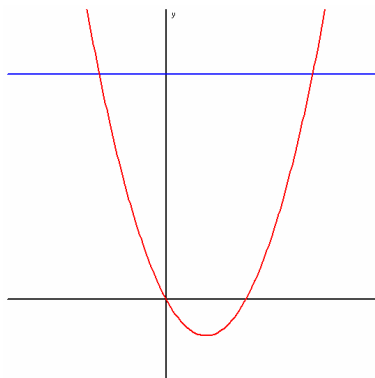
b. $y_1 = x^2 + 2x$ en $y_2 = 11$
met de optie intersect:
$x = -4,46$ $\lor$ $x = 2,46$

$-2,53 \leq x \leq 5,53$
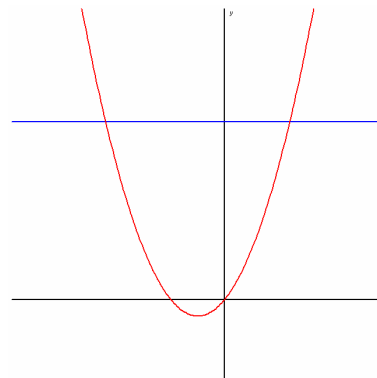
$x < -4,46$ $\lor$ $x > 2,46$

c. $y_1 = 8x^2 + 6x - 35$

met de optie zero:

$x = -2,5 \quad \vee \quad x = 1,75$

$x = -6 \quad \vee \quad x = -2,5 \quad \vee \quad x = 4$
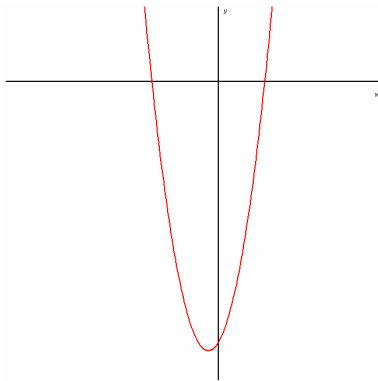
$x \leq -2,5 \quad \vee \quad x \geq 1,75$

d. $y_1 = x^3 + 4,5x^2$ en $y_2 = 19x + 60$

met de optie intersect:
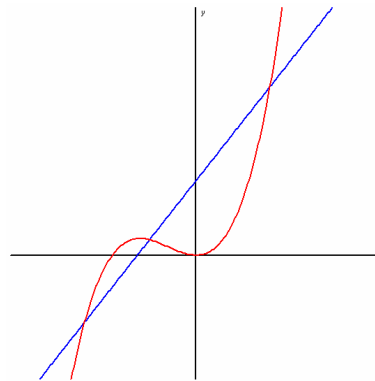
$x < -6 \quad \vee \quad -2,5 < x < 4$

Opgave 68:

a. $x^2 - 5x < 14$

$x^2 - 5x - 14 = 0$

$(x - 7)(x + 2) = 0$

$x = 7 \quad \vee \quad x = -2$

$-2 < x < 7$
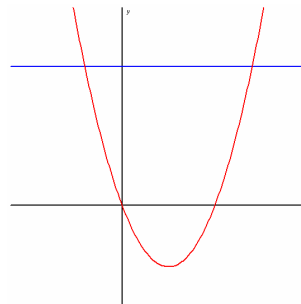
b. $2x^2 - 3x \geq 2$

$2x^2 - 3x - 2 = 0$

$x = \dfrac{3 \pm \sqrt{9 + 16}}{4} = \dfrac{3 \pm 5}{4}$

$x = \dfrac{3 + 5}{4} = 2 \quad \vee \quad x = \dfrac{3 - 5}{4} = -\tfrac{1}{2}$

$x \leq -\tfrac{1}{2} \quad \vee \quad x \geq 2$
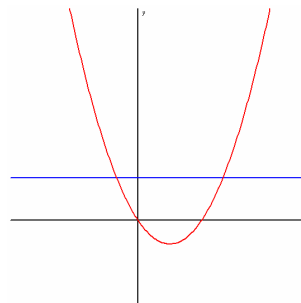
c. $x^2 - 4x \leq -x^2 - 5x + 6$

$2x^2 + x - 6 = 0$

$x = \dfrac{-1 \pm \sqrt{1 + 48}}{4} = \dfrac{-1 \pm 7}{4}$

$x = \dfrac{-1 + 7}{4} = 1\tfrac{1}{2} \quad \vee \quad x = \dfrac{-1 - 7}{4} = -2$

$-2 \leq x \leq 1\tfrac{1}{2}$
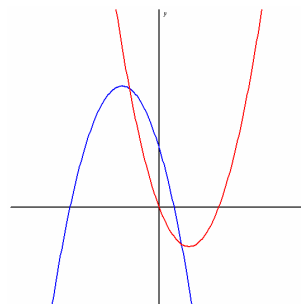
d. $x^3 + 2x^2 > 3x$

    $x^3 + 2x^2 - 3x = 0$

    $x(x^2 + 2x - 3) = 0$

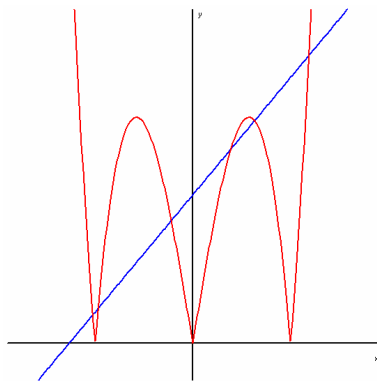 $x(x + 3)(x - 1) = 0$

    $x = 0 \quad \vee \quad x = -3 \quad \vee \quad x = 1$

    $-3 < x < 0 \quad \vee \quad x > 1$
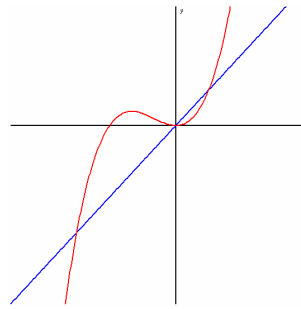
Opgave 69:

a. $y_1 = 0{,}1x^3 - 2x^2 + 8x + 10$

    $y_2 = -x + 15$

    met de optie intersection:
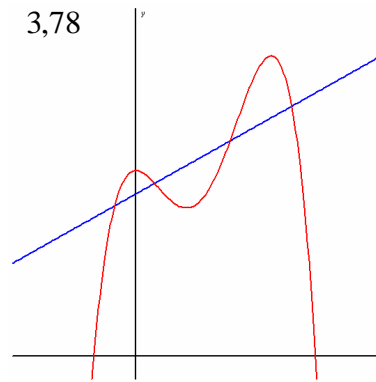
    $x = 0{,}65 \quad \vee \quad x = 5{,}66 \quad \vee \quad x = 13{,}69$

    $0{,}65 \leq x \leq 5{,}66 \quad \vee \quad x \geq 13{,}69$

b. $y_1 = -0{,}5x^4 + 3x^3 - 4x^2 + 8$

    $y_2 = x + 7$

    met de optie intersection:

    $x = -0{,}52 \quad \vee \quad x = 0{,}45 \quad \vee \quad x = 2{,}29$

    $\vee \quad 3{,}78$
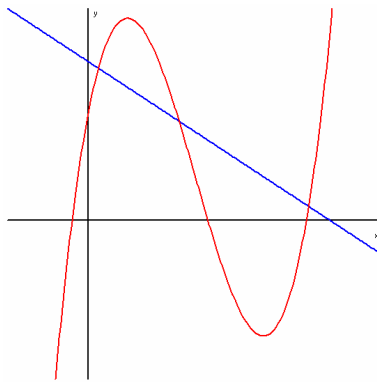
    $-0{,}52 \leq x \leq 0{,}45 \quad \vee \quad 2{,}29 \leq x \leq 3{,}78$

c. $y_1 = abs(x^3 - 10x)$

    $y_2 = 2x + 8$

    met de optie intersection:

    $x = -3{,}24 \quad \vee \quad x = -3{,}06 \quad \vee \quad x = -0{,}69$

    $\vee \quad x = 1{,}24 \quad \vee \quad x = 2 \quad \vee \quad x = 3{,}76$

    $-3{,}24 \leq x \leq -3{,}06 \quad \vee \quad -0{,}69 \leq x \leq 1{,}24$

    $\vee \quad 2 \leq x \leq 3{,}76$

d. $y_1 = abs(x^4 + x^2 - 5x - 10)$

    $y_2 = 8 - abs(2x - 4)$

    met de optie intersection:

    $x = -1{,}32 \quad \vee \quad x = -1{,}10 \quad \vee \quad x = 1{,}69$

    $\vee \quad x = 2{,}21$
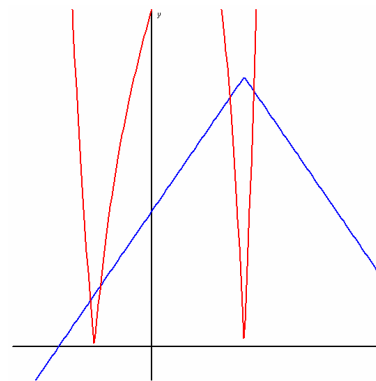
    $-1{,}32 \leq x \leq -1{,}10 \quad \vee \quad 1{,}69 \leq x \leq 2{,}21$

Opgave 70:

a. $D = p^2 - 4p$

b. $p^2 - 4p > 0$

Opgave 71:

a. $D = p^2 - 12p > 0$

    $p(p - 12) = 0$

    $p = 0 \quad \vee \quad p = 12$

    $p < 0 \quad \vee \quad p > 12$

b. $p \neq 0 \quad \wedge \quad D = (p - 4)^2 - 2p > 0$

    $p^2 - 8p + 16 - 2p > 0$

    $p^2 - 10p + 16 > 0$

    $(p - 2)(p - 8) = 0$

    $p = 2 \quad \vee \quad p = 8$

    $p < 2 \quad \vee \quad p > 8$

    dus $p < 0 \quad \vee \quad 0 < p < 2 \quad \vee \quad p > 8$
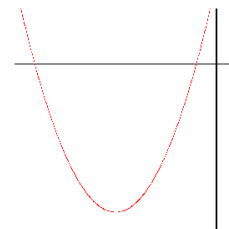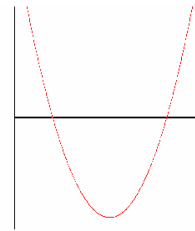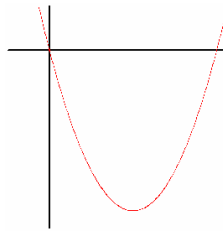
c.   dus $px^2 + (p - 3)x - 4 = 0$ heeft geen oplossingen

    $p \neq 0 \quad \wedge \quad D = (p - 3)^2 + 16p < 0$

    $p^2 - 6p + 9 + 16p < 0$

    $p^2 + 10p + 9 < 0$

    $(p - 1)(p - 9) = 0$

    $p = 1 \quad \vee \quad p = 9$

    $1 < p < 9$

Opgave 72:

a. $D = (p^2 - 2)^2 - 49 > 0$

    $(p^2 - 2)^2 = 49$

    $p^2 - 2 = 7 \quad \vee \quad p^2 - 2 = -7$

    $p^2 = 9 \quad \vee \quad p^2 = -5$

    $p = 3 \quad \vee \quad p = -3$

    $p < -3 \quad \vee \quad p > 3$

b. $px^3 + p^2x^2 - 16x = 0$

    $x(px^2 + p^2x - 16) = 0$

    $x = 0 \quad \vee \quad px^2 + p^2x - 16 = 0$ heeft 2 oplossingen

    $D = p^4 + 64p > 0$

    $p(p^3 + 64) = 0$

    $p = 0 \quad \vee \quad p^3 = -64$

    $p = 0 \quad \vee \quad p = -4$

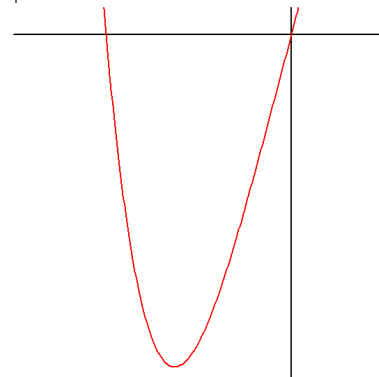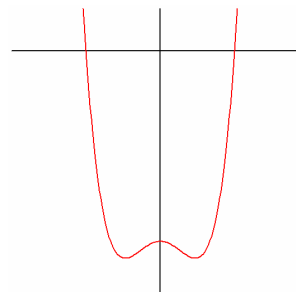 $p < -4 \quad \vee \quad p > 0$

c. $px^3 + 2px^2 - 3x^2 + \frac{1}{4}x = 0$

    $x(px^2 + (2p - 3)x + \frac{1}{4}) = 0$

    $x = 0 \quad \vee \quad px^2 + (2p - 3) + \frac{1}{4} = 0$ heeft geen oplossingen

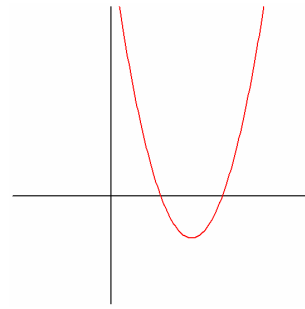 $D = (2p - 3)^2 - p < 0$

    $4p^2 - 12p + 9 - p < 0$

$4p^2 - 13p + 9 < 0$

$p = \dfrac{13 \pm \sqrt{169 - 144}}{8} = \dfrac{13 \pm 5}{8}$

$p = \dfrac{13 + 5}{8} = 2\tfrac{1}{4} \quad \vee \quad p = \dfrac{13 - 5}{8} = 1$

$1 < p < 2\tfrac{1}{4}$

Diagnostische toets hoofdstuk 1.

Opgave 1:
a. $3x^2 - x = 0$
     $x(3x - 1) = 0$
     $x = 0 \quad \vee \quad 3x = 1$
     $x = 0 \quad \vee \quad x = \frac{1}{3}$

b. $3x^2 - 9x = 12$
     $3x^2 - 9x - 12 = 0$
     $x^2 - 3x - 4 = 0$
     $(x - 4)(x + 1) = 0$
     $x = 4 \quad \vee \quad x = -1$

c. $3x^2 - x = 2$
     $3x^2 - x - 2 = 0$
     $x = \dfrac{1 \pm \sqrt{1 + 24}}{6} = \dfrac{1 \pm 5}{6}$
     $x = \dfrac{1 + 5}{6} = 1 \quad \vee \quad x = \dfrac{1 - 5}{6} = -\frac{2}{3}$

d. $x^2 + 14 = 16$
     $x^2 = 2$
     $x = \sqrt{2} \quad \vee \quad x = -\sqrt{2}$

e. $(2x - 3)^2 = 81$
     $2x - 3 = 9 \quad \vee \quad 2x - 3 = -9$
     $2x = 12 \quad \vee \quad 2x = -6$
     $x = 6 \quad \vee \quad x = -3$

f. $(3x + 2)(x - 1) = 0$
     $3x = -2 \quad \vee \quad x = 1$
     $x = -\frac{2}{3} \quad \vee \quad x = 1$

g. $x^2 = 7x + 13$
     $x^2 - 7x - 13 = 0$
     $x = \dfrac{7 \pm \sqrt{49 + 52}}{2} = \dfrac{7 \pm \sqrt{101}}{2}$
     $x = \dfrac{7 + \sqrt{101}}{2} \quad \vee \quad x = \dfrac{7 - \sqrt{101}}{2}$

h. $(3x + 2)(x - 1) = (x + 5) \cdot x$
     $3x^2 - 3x + 2x - 2 = x^2 + 5x$
     $2x^2 - 6x - 2 = 0$
     $x^2 - 3x - 1 = 0$
     $x = \dfrac{3 \pm \sqrt{9 + 4}}{2} = \dfrac{3 \pm \sqrt{13}}{2}$
     $x = \dfrac{3 + \sqrt{13}}{2} \quad \vee \quad x = \dfrac{3 - \sqrt{13}}{2}$
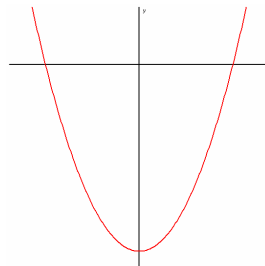
i. $(x+2)^2 = 3x+7$

$x^2 + 4x + 4 = 3x + 7$

$x^2 + x - 3 = 0$

$x = \dfrac{-1 \pm \sqrt{1+12}}{2} = \dfrac{-1 \pm \sqrt{13}}{2}$

$x = \dfrac{-1 + \sqrt{13}}{2} \quad \vee \quad x = \dfrac{-1 - \sqrt{13}}{2}$

j. $(x-3)^2 - (x+1)^2 = (x-4)^2$

$x^2 - 6x + 9 - (x^2 + 2x + 1) = x^2 - 8x + 16$

$x^2 - 6x + 9 - x^2 - 2x - 1 - x^2 + 8x - 16 = 0$

$-x^2 - 8 = 0$

$-x^2 = 8$

$x^2 = -8$ dus geen oplossingen

Opgave 2:

a. $D = 4^2 - 4 \cdot 2 \cdot p = 16 - 8p < 0$

$-8p < -16$

$p > 2$

b. $D = p^2 - 4 \cdot 3 \cdot 27 = p^2 - 324 > 0$

$p^2 > 324$

$p = 18 \quad \vee \quad p = -18$

$p < -18 \quad \vee \quad p > 18$

c. als $p = 0$ heb je een eerstegraads vergelijking

$2x + 5 = 0$

$2x = -5$

$x = -2\tfrac{1}{2}$

als $p \neq 0$ heb je een tweedegraads vergelijking dus moet gelden $D = 0$

$D = (-6)^2 - 4 \cdot p \cdot 12 = 36 - 48p = 0$

$-48p = -36$

$p = \tfrac{3}{4}$

$\tfrac{3}{4} x^2 - 6x + 12 = 0$

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

$x = 4$

dus één oplossing als $p = 0 \quad \vee \quad p = \tfrac{3}{4}$

Opgave 3:

a. $x = 2$ invullen geeft $4 + 8 + p = 0$

dus $p = -12$

$x^2 + 4x - 12 = 0$

$(x+6)(x-2) = 0$

$x = -6 \quad \vee \quad x = 2$

b. $p \neq 0 \quad \wedge \quad D = 2^2 - 4 \cdot p \cdot 5 = 4 - 20p > 0$

$p \neq 0 \quad \wedge \quad -20p > -4$

$p \neq 0 \quad \wedge \quad p < \frac{1}{5}$

$p < 0 \quad \vee \quad 0 < p < \frac{1}{5}$

Opgave 4:

a. $3x^3 + 5 = 86$

$3x^3 = 81$

$x^3 = 27$

$x = \sqrt[3]{27} = 3$

b. $5x^4 - 6 = 9$

$5x^4 = 15$

$x^4 = 3$

$x = \sqrt[4]{3} \quad \vee \quad x = -\sqrt[4]{3}$

c. $2x^3 + 19 = 5$

$2x^3 = -14$

$x^3 = -7$

$x = \sqrt[3]{-7}$

d. $\frac{1}{2}(x+2)^4 = \frac{1}{32}$

$(x+2)^4 = \frac{1}{16}$

$x + 2 = \sqrt[4]{\frac{1}{16}} = \frac{1}{2} \quad \vee \quad x + 2 = -\sqrt[4]{\frac{1}{16}} = -\frac{1}{2}$

$x = -1\frac{1}{2} \quad \vee \quad x = -2\frac{1}{2}$

e. $100 - (2x+1)^5 = 68$

$-(2x+1)^5 = -32$

$(2x+1)^5 = 32$

$2x + 1 = \sqrt[5]{32} = 2$

$2x = 1$

$x = \frac{1}{2}$

f. $(2x+4)^3 = 10$

$2x + 4 = \sqrt[3]{10}$

$2x = -4 + \sqrt[3]{10}$

$x = -2 + \frac{1}{2} \cdot \sqrt[3]{10}$

Opgave 5:

a. $x^4 - 6x^2 + 5 = 0$

stel $x^2 = p$

$p^2 - 6p + 5 = 0$

$(p-1)(p-5) = 0$

$p = 1 \quad \vee \quad p = 5$

$x^2 = 1 \quad \vee \quad x^2 = 5$

$x = -1 \quad \vee \quad x = 1 \quad \vee \quad x = -\sqrt{5} \quad \vee \quad x = \sqrt{5}$

b. $5x^4 - 6x^2 + 1 = 0$

stel $x^2 = p$

$5p^2 - 6p + 1 = 0$

$p = \dfrac{6 \pm \sqrt{36 - 20}}{10} = \dfrac{6 \pm 4}{10}$

$p = \dfrac{6+4}{10} = 1 \quad \vee \quad p = \dfrac{6-4}{10} = \tfrac{1}{5}$

$x^2 = 1 \quad \vee \quad x^2 = \tfrac{1}{5}$

$x = 1 \quad \vee \quad x = -1 \quad \vee \quad x = \sqrt{\tfrac{1}{5}} \quad \vee \quad x = -\sqrt{\tfrac{1}{5}}$

c. $x^4 - 6x^3 + 5x^2 = 0$

$x^2(x^2 - 6x + 5) = 0$

$x^2(x-1)(x-5) = 0$

$x = 0 \quad \vee \quad x = 1 \quad \vee \quad x = 5$

d. $x^3 + 6x^2 + 2x = 0$

$x(x^2 + 6x + 2) = 0$

$x = 0 \quad \vee \quad x^2 + 6x + 2 = 0$

$x = 0 \quad \vee \quad x = \dfrac{-6 \pm \sqrt{36 - 8}}{2}$

$x = 0 \quad \vee \quad x = \dfrac{-6 + \sqrt{24}}{2} \quad \vee \quad x = \dfrac{-6 - \sqrt{24}}{2}$

e. $3x^6 + 3 = 10x^3$

$3x^6 - 10x^3 + 3 = 0$

stel $x^3 = p$

$3p^2 - 10p + 3 = 0$

$p = \dfrac{10 \pm \sqrt{100 - 36}}{6} = \dfrac{10 \pm 8}{6}$

$p = \dfrac{10+8}{6} = 3 \quad \vee \quad p = \dfrac{10-8}{6} = \tfrac{1}{3}$

$x^3 = 3 \quad \vee \quad x^3 = \tfrac{1}{3}$

$x = \sqrt[3]{3} \quad \vee \quad x = \sqrt[3]{\tfrac{1}{3}}$

f. $x^8 + x^4 = 42$

$x^8 + x^4 - 42 = 0$

stel $x^4 = p$

$p^2 + p - 42 = 0$

$(p+7)(p-6) = 0$

$p = -7 \quad \vee \quad p = 6$

$x^4 = -7 \quad \vee \quad x^4 = 6$

$x = \sqrt[4]{6} \quad \vee \quad x = -\sqrt[4]{6}$

Opgave 6:

a. $\left|x^2 - 4\right| = 21$

$x^2 - 4 = 21 \quad \vee \quad x^2 - 4 = -21$

$x^2 = 25 \quad \vee \quad x^2 = -17$

$x = 5 \quad \vee \quad x = -5$

b. $\left|4x^3 - 5\right| = 17$

$4x^3 - 5 = 17 \quad \vee \quad 4x^3 - 5 = -17$

$4x^3 = 22 \quad \vee \quad 4x^3 = -12$

$x^3 = 5\tfrac{1}{2} \quad \vee \quad x^3 = -3$

$x = \sqrt[3]{5\tfrac{1}{2}} \quad \vee \quad x = \sqrt[3]{-3}$

Opgave 7:

a. $\sqrt{3x + 5} + 1 = 5$

$\sqrt{3x + 5} = 4$

$3x + 5 = 16$

$3x = 11$

$x = 3\tfrac{2}{3}$

b. $3x = 5\sqrt{x + 4}$

$9x^2 = 25(x + 4)$

$9x^2 = 25x + 100$

$9x^2 - 25x - 100 = 0$

$x = \dfrac{25 \pm \sqrt{625 + 3600}}{18} = \dfrac{25 \pm \sqrt{4225}}{18} = \dfrac{25 \pm 65}{18}$

$x = \dfrac{25 + 65}{18} = 5 \quad \vee \quad x = \dfrac{25 - 65}{18} = -2\tfrac{2}{9} \text{ (vervalt)}$

c. $x = \sqrt{x} + 6$

$x - 6 = \sqrt{x}$

$x^2 - 12x + 36 = x$

$x^2 - 13x + 36 = 0$

$(x - 4)(x - 9) = 0$

$x = 4 \text{ (vervalt)} \quad \vee \quad x = 9$

d. $2x + 3\sqrt{x} = 2$

$3\sqrt{x} = 2 - 2x$

$9x = 4 - 8x + 4x^2$

$-4x^2 + 17x - 4 = 0$

$x = \dfrac{-17 \pm \sqrt{289 - 64}}{-8} = \dfrac{-17 \pm \sqrt{225}}{-8} = \dfrac{-17 \pm 15}{-8}$

$x = \dfrac{-17 - 15}{-8} = 4 \text{ (vervalt)} \quad \vee \quad x = \dfrac{-17 + 15}{-8} = \tfrac{1}{4}$

Opgave 8:

a. $x^3 - 189 = 20x\sqrt{x}$

$x^3 - 20x\sqrt{x} - 189 = 0$

stel $x\sqrt{x} = p$

$p^2 - 20p - 189 = 0$

$(p - 27)(p + 7) = 0$

$p = 27 \quad \vee \quad p = -7$

$x\sqrt{x} = 27 \quad \vee \quad x\sqrt{x} = -7$

$x^3 = 729 \quad \vee \quad x^3 = 49$

$x = \sqrt[3]{729} = 9 \quad \vee \quad x = \sqrt[3]{49}$ (vervalt)

b. $x^5 + 12 = 8x^2 \cdot \sqrt{x}$

$x^5 - 8x^2 \cdot \sqrt{x} + 12 = 0$

stel $x^2 \cdot \sqrt{x} = p$

$p^2 - 8p + 12 = 0$

$(p - 6)(p - 2) = 0$

$p = 6 \quad \vee \quad p = 2$

$x^2 \cdot \sqrt{x} = 6 \quad \vee \quad x^2 \cdot \sqrt{x} = 2$

$x^5 = 36 \quad \vee \quad x^5 = 4$

$x = \sqrt[5]{36} \quad \vee \quad x = \sqrt[5]{4}$

Opgave 9:

a. $\dfrac{6x - 18}{x + 1} = 0$

$6x - 18 = 0$

$6x = 18$

$x = 3$

b. $\dfrac{x^2 - 5x + 6}{2x + 4} = 0$

$x^2 - 5x + 6 = 0$

$(x - 2)(x - 3) = 0$

$x = 2 \quad \vee \quad x = 3$

c. $\dfrac{3x - 5}{x + 1} = \dfrac{x + 2}{x + 1}$

$3x - 5 = x + 2$

$2x = 7$

$x = 3\tfrac{1}{2}$

d. $\dfrac{x^2 - 4}{2x + 1} = \dfrac{x^2 - 4}{x - 4}$

$2x + 1 = x - 4$

$x = -5$

e. $\dfrac{2x-1}{x+1} = \dfrac{x+3}{x-4}$

$(2x-1)(x-4) = (x+1)(x+3)$

$2x^2 - 8x - x + 4 = x^2 + 3x + x + 3$

$x^2 - 13x + 1 = 0$

$x = \dfrac{13 \pm \sqrt{169-4}}{2} = \dfrac{13 \pm \sqrt{165}}{2}$

$x = \dfrac{13 + \sqrt{165}}{2} \quad \lor \quad x = \dfrac{13 - \sqrt{165}}{2}$

f. $\dfrac{2x^2-4}{x+5} = 1\tfrac{3}{4}$

$\dfrac{2x^2-4}{x+5} = \dfrac{7}{4}$

$4(2x^2 - 4) = 7(x+5)$

$8x^2 - 16 = 7x + 35$

$8x^2 - 7x - 51 = 0$

$x = \dfrac{7 \pm \sqrt{49+1632}}{16} = \dfrac{7 \pm \sqrt{1681}}{16} = \dfrac{7 \pm 41}{16}$

$x = \dfrac{7+41}{16} = 3 \quad \lor \quad x = \dfrac{7-41}{16} = -2\tfrac{1}{8}$

Opgave 10:

a. $\begin{cases} 4x+5y=27 & |\times 1 \\ -2x+3y=25 & |\times 2 \end{cases}$

$\begin{cases} 4x+5y=27 \\ -4x+6y=50 \quad + \end{cases}$

$\overline{\qquad 11y=77}$

$y=7$

$4x+35=27$

$4x=-8$

$x=-2$

$x=-2 \quad \land \quad y=7$

b. $\begin{cases} 2x+3y=7 & |\times 2 \\ 5x-2y=8 & |\times 3 \end{cases}$

$\begin{cases} 4x+6y=14 \\ 15x-6y=24 \quad + \end{cases}$

$\overline{\qquad 19x=38}$

$x=2$

$4+3y=7$

$3y=3$

$y=1$

$x=2 \quad \land \quad y=1$

Opgave 11:

$$\begin{cases} 4a + 2b = 18 & |\times 2 \\ 16a - 4b = 0 & |\times 1 \end{cases}$$

$$\begin{cases} 8a + 4b = 36 \\ 16a - 4b = 0 \quad + \end{cases}$$
$$\overline{\qquad 24a = 36 \qquad}$$

$a = 1\frac{1}{2}$
$6 + 2b = 18$
$2b = 12$
$b = 6$
$y = 1\frac{1}{2}x^2 + 6x$

Opgave 12:

a. $5x - 3y = 3 \quad \wedge \quad y = \frac{2}{3}x - 4$

  $5x - 3(\frac{2}{3}x - 4) = 3$

  $5x - 2x + 12 = 3$

  $3x = -9$

  $x = -3$

  $y = -6$

  dus $x = -3 \quad \wedge \quad y = -6$

b. $2x + 3y = 10 \quad \wedge \quad y = x^2 - 4x + 6$

  $2x + 3(x^2 - 4x + 6) = 10$

  $2x + 3x^2 - 12x + 18 = 10$

  $3x^2 - 10x + 8 = 0$

  $x = \dfrac{10 \pm \sqrt{100 - 96}}{6} = \dfrac{10 \pm 2}{6}$

  $x = \dfrac{10 + 2}{6} = 2 \quad \vee \quad x = \dfrac{10 - 2}{6} = 1\frac{1}{3}$

  $y = 2 \qquad\qquad \vee \quad y = 2\frac{4}{9}$

  $(x = 2 \quad \wedge \quad y = 2) \quad \vee \quad (x = 1\frac{1}{3} \quad \wedge \quad y = 2\frac{4}{9})$

Opgave 13:

a. $y_1 = x^4 - 4x^2$

  $y_2 = 0{,}5x - 2$

  De optie intersection geeft: $x = -1{,}75 \quad \vee \quad x = -0{,}86 \quad \vee \quad x = 0{,}69 \quad \vee \quad x = 1{,}93$

b. $y_1 = abs(x^3 - 3x)$

  $y_2 = -\frac{1}{2}x + 2$

  De optie intersection geeft: $x = -2{,}11 \quad \vee \quad x = 0{,}65 \quad \vee \quad x = 1{,}89 \quad \vee \quad x = 1{,}46$

Opgave 14:
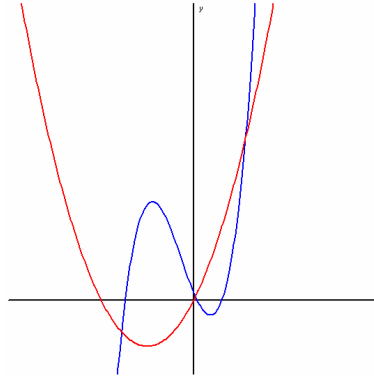
a. $y_1 = x^2 + 5x$

   $y_2 = x^3 + 2x^2 - 6x + 1$

   De optie intersection geeft:

   $x = -3,89 \quad \vee \quad x = 0,09 \quad \vee \quad x = 2,80$

   $-3,89 \le x \le 0,09 \quad \vee \quad x \ge 2,80$

b. $y_1 = abs(x^2 - 4x)$

   $y_2 = \frac{1}{2}x + 2$

   De optie intersection geeft:

   $x = -0,41 \quad \vee \quad x = 0,72 \quad \vee \quad x = 2,78 \quad \vee \quad x = 4,91$

   $x < -0,41 \quad \vee \quad 0,72 < x < 2,78 \quad \vee \quad x > 4,91$
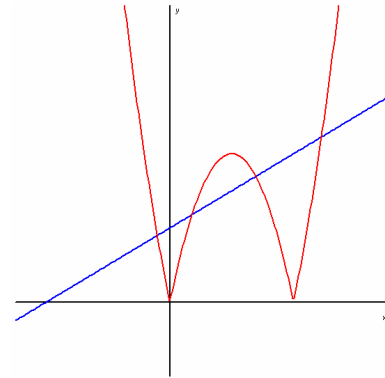
Opgave 15:

a. $3x^2 + 2x \ge 33$

   $3x^2 + 2x - 33 = 0$

   $x = \dfrac{-2 \pm \sqrt{4 + 396}}{6} = \dfrac{-2 \pm 20}{6}$

   $x = \dfrac{-2 + 20}{6} = 3 \quad \vee \quad x = \dfrac{-2 - 20}{6} = -3\tfrac{2}{3}$
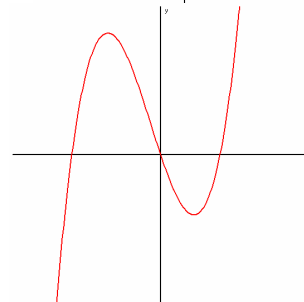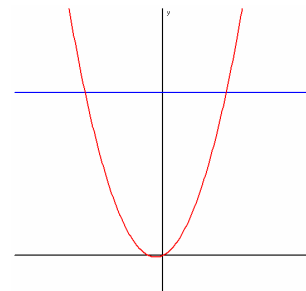
   $x \le -3\tfrac{2}{3} \quad \vee \quad x \ge 3$

b. $x^3 + x^2 - 6x < 0\ \text{x}$

   $x(x^2 + x - 6) = 0$

   $x(x + 3)(x - 2) = 0$

   $x = 0 \quad \vee \quad x = -3 \quad \vee \quad x = 2$

   $x < -3 \quad \vee \quad 0 < x < 2$

Opgave 16:

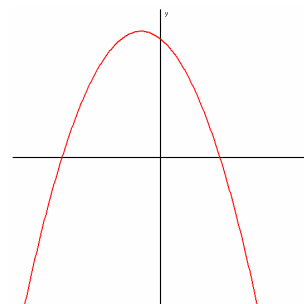a. $D = (p - 2)^2 - 4 \cdot p \cdot 4p < 0$

   $p^2 - 4p + 4 - 16p^2 < 0$

   $-15p^2 - 4p + 4 < 0$

   $p = \dfrac{4 \pm \sqrt{16 + 240}}{-30} = \dfrac{4 \pm 16}{-30}$

   $p = \dfrac{4 + 16}{-30} = -\tfrac{2}{3} \quad \vee \quad p = \dfrac{4 - 16}{-30} = \tfrac{2}{5}$

   $p < -\tfrac{2}{3} \quad \vee \quad p > \tfrac{2}{5}$

b. $x(px^2 + 2px - 2) = 0$

$x = 0 \quad \vee \quad px^2 + 2px - 2 = 0$ heeft geen oplossingen

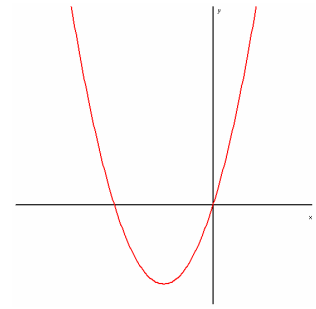$D = (2p)^2 - 4 \cdot p \cdot -2 < 0$

$4p^2 + 8p < 0$

$4p(p + 2) = 0$

$p = 0 \quad \vee \quad p = -2$

$-2 < p < 0$

voor $p = 0$ heb je de eerstegraads vergelijking $-2x = 0$ dus $x = 0$

dus $-2 < p \le 0$

GEMENGDE OPGAVEN: H1 Vergelijkingen en ongelijkheden.

Opgave 1:
a. $7x^2 = 5x$

$7x^2 - 5x = 0$

$x(7x - 5) = 0$

$x = 0 \quad \vee \quad 7x = 5$

$x = 0 \quad \vee \quad x = \frac{5}{7}$

b. $2x^2 + x = 3$

$2x^2 + x - 3 = 0$

$x = \frac{-1 \pm \sqrt{1 + 24}}{4} = \frac{-1 \pm 5}{4}$

$x = \frac{-1 + 5}{4} = 1 \quad \vee \quad x = \frac{-1 - 5}{4} = -1\frac{1}{2}$

c. $(x + 2)(x - 6) = 9$

$x^2 - 4x - 12 = 9$

$x^2 - 4x - 21 = 0$

$(x - 7)(x + 3) = 0$

$x = 7 \quad \vee \quad x = -3$

d. $(x - 3)^2 - (x + 1)^2 = x^2 - 1$

$x^2 - 6x + 9 - (x^2 + 2x + 1) = x^2 - 1$

$x^2 - 6x + 9 - x^2 - 2x - 1 = x^2 - 1$

$-x^2 - 8x + 9 = 0$

$x^2 + 8x - 9 = 0$

$(x + 9)(x - 1) = 0$

$x = -9 \quad \vee \quad x = 1$

e. $(2x - 3)^2 = 36$

$2x - 3 = 6 \quad \vee \quad 2x - 3 = -6$

$2x = 9 \quad \vee \quad 2x = -3$

$x = 4\frac{1}{2} \quad \vee \quad x = -1\frac{1}{2}$

f. $4 - (x - 2)^2 = 7x - 3$

$4 - (x^2 - 4x + 4) = 7x - 3$

$4 - x^2 + 4x - 4 = 7x - 3$

$-x^2 - 3x + 3 = 0$

$x = \frac{3 \pm \sqrt{9 + 12}}{-2} = -1\frac{1}{2} \pm \frac{1}{2}\sqrt{21}$

$x = -1\frac{1}{2} + \frac{1}{2}\sqrt{21} \quad \vee \quad x = -1\frac{1}{2} - \frac{1}{2}\sqrt{21}$

Opgave 2:
a. $D = 6^2 - 4 \cdot p \cdot 3p > 0$

$36 - 12p^2 > 0$

$-12p^2 > -36$

$p^2 < 3$

$p = \sqrt{3} \quad \vee \quad p = -\sqrt{3}$

$-\sqrt{3} < x < \sqrt{3}$

b. $36 + 6p - 6p^2 = 0$

$p^2 - p - 6 = 0$

$(p - 3)(p + 2) = 0$

$p = 3 \quad \vee \quad p = -2$

$x^2 + 3x - 54 = 0 \quad \vee \quad x^2 - 2x - 24 = 0$

$(x + 9)(x - 6) = 0 \quad \vee \quad (x - 6)(x + 4) = 0$

$x = -9 \quad \vee \quad x = 6 \quad \vee \quad x = 6 \quad \vee \quad x = -4$

c. $D = (-2p)^2 - 4 \cdot p \cdot 4 = 0 \quad \wedge \quad p \neq 0$ (als $p = 0$ dan $4 = 0$ dus geen oplossingen)

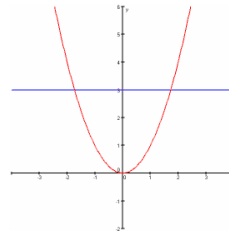$4p^2 - 16p = 0$

$4p(p - 4) = 0$

$p = 0 \quad \vee \quad p = 4$

k.n. $4x^2 - 8x + 4 = 0$

$x^2 - 2x + 1 = 0$

$(x - 1)^2 = 0$

$x = 1$

Opgave 3:

a. $x^6 - 6x^3 + 5 = 0$

stel $p = x^3$ dan $p^2 - 6p + 5 = 0$

$(p - 1)(p - 5) = 0$

$p = 1 \quad \vee \quad p = 5$

$x^3 = 1 \quad \vee \quad x^3 = 5$

$x = 1 \quad \vee \quad x = \sqrt[3]{5}$

b. $\left| x^4 - 7x^2 \right| = 18$

$x^4 - 7x^2 = 18 \quad \vee \quad x^4 - 7x^2 = -18$

stel $p = x^2$

$p^2 - 7p = 18 \quad \vee \quad p^2 - 7p = -18$

$p^2 - 7p - 18 = 0 \quad \vee \quad p^2 - 7p + 18 = 0$

$(p + 2)(p - 9) = 0 \quad \vee \quad p = \dfrac{7 \pm \sqrt{49 - 72}}{2} = k.n.$

$p = -2 \quad \vee \quad p = 9$

$x^2 = -2 \quad \vee \quad x^2 = 9$

k.n. $x = 3 \quad \vee \quad x = -3$

c. $10x^4 = 17x^2 + 657$

$10x^4 - 17x^2 - 657 = 0$

stel $p = x^2$

$10p^2 - 17p - 657 = 0$

$$p = \frac{17 \pm \sqrt{289 + 26280}}{20} = \frac{17 \pm \sqrt{26569}}{20} = \frac{17 \pm 163}{20}$$

$$p = \frac{17 + 163}{20} = 9 \quad \vee \quad p = \frac{17 - 163}{20} = -7{,}3$$

$$x^2 = 9 \quad \vee \quad x^2 = -7{,}3$$

$$x = 3 \quad \vee \quad x = -3$$

d. $10 - (2x-1)^4 = 8$

$-(2x-1)^4 = -2$

$(2x-1)^4 = 2$

$2x - 1 = \sqrt[4]{2} \quad \vee \quad 2x - 1 = -\sqrt[4]{2}$

$2x = 1 + \sqrt[4]{2} \quad \vee \quad 2x = 1 - \sqrt[4]{2}$

$x = \tfrac{1}{2} + \tfrac{1}{2} \cdot \sqrt[4]{2} \quad \vee \quad x = \tfrac{1}{2} - \tfrac{1}{2} \cdot \sqrt[4]{2}$

e. $x^5 - 16x^3 + 28x = 0$

$x(x^4 - 16x^2 + 28) = 0$

$x = 0 \quad \vee \quad x^4 - 16x^2 + 28 = 0$

stel $p = x^2$

$x = 0 \quad \vee \quad p^2 - 16p + 28 = 0$

$x = 0 \quad \vee \quad (p-2)(p-14) = 0$

$x = 0 \quad \vee \quad p = 2 \quad \vee \quad p = 14$

$x = 0 \quad \vee \quad x^2 = 2 \quad \vee \quad x^2 = 14$

$x = 0 \quad \vee \quad x = \sqrt{2} \quad \vee \quad x = -\sqrt{2} \quad \vee \quad x = \sqrt{14} \quad \vee \quad x = -\sqrt{14}$

f. $x^3 - 3x\sqrt{x} - 108 = 0$

stel $p = x\sqrt{x}$

$p^2 - 3p - 108 = 0$

$(p+9)(p-12) = 0$

$p = -9 \quad \vee \quad p = 12$

$x\sqrt{x} = -9 \quad \vee \quad x\sqrt{x} = 12$

$\quad$ k.n. $\qquad\qquad x^3 = 144$

$\qquad\qquad\qquad\quad x = \sqrt[3]{144}$

g. $6x^5 + 10x^2 \cdot \sqrt{x} - 464 = 0$

stel $p = x^2 \cdot \sqrt{x}$

$6p^2 + 10p - 464 = 0$

$$p = \frac{-10 \pm \sqrt{100 + 11136}}{12} = \frac{-10 \pm \sqrt{11236}}{12} = \frac{-10 \pm 106}{12}$$

$$p = \frac{-10 - 106}{12} = -9\tfrac{2}{3} \quad \vee \quad p = \frac{-10 + 106}{12} = 8$$

$x^2 \cdot \sqrt{x} = -9\tfrac{2}{3} \quad \vee \quad x^2 \cdot \sqrt{x} = 8$

$\quad$ k.n. $\qquad\qquad x^5 = 64$

$\qquad\qquad\qquad\quad x = \sqrt[5]{64}$

h. $(2x-1)^4 - 5(2x-1)^2 + 4 = 0$

stel $p = 2x - 1$

$p^4 - 5p^2 + 4 = 0$

$(p^2 - 1)(p^2 - 4) = 0$

$p^2 = 1 \quad \vee \quad p^2 = 4$

$p = 1 \quad \vee \quad p = -1 \quad \vee \quad p = 2 \quad \vee \quad p = -2$

$2x - 1 = 1 \quad \vee \quad 2x - 1 = -1 \quad \vee \quad 2x - 1 = 2 \quad \vee \quad 2x - 1 = -2$

$2x = 2 \quad \vee \quad 2x = 0 \quad \vee \quad 2x = 3 \quad \vee \quad 2x = -1$

$x = 1 \quad \vee \quad x = 0 \quad \vee \quad x = 1\tfrac{1}{2} \quad \vee \quad x = -\tfrac{1}{2}$

Opgave 4:

a. $\dfrac{2x}{x-1} - 2 = 4$

$\dfrac{2x}{x-1} = 6$

$6(x-1) = 2x$

$6x - 6 = 2x$

$4x = 6$

$x = 1\tfrac{1}{2}$

b. $3\sqrt{2-3x} = 21$

$\sqrt{2-3x} = 7$

$2 - 3x = 49$

$-3x = 47$

$x = -15\tfrac{2}{3}$

c. $\dfrac{x+2}{x-1} = \dfrac{x}{x+5}$

$(x+2)(x+5) = x(x-1)$

$x^2 + 7x + 10 = x^2 - x$

$8x = -10$

$x = -1\tfrac{1}{4}$

d. $\dfrac{2x-1}{x+2} = \dfrac{x+4}{x-2}$

$(2x-1)(x-2) = (x+2)(x+4)$

$2x^2 - 5x + 2 = x^2 + 6x + 8$

$x^2 - 11x - 6 = 0$

$x = \dfrac{11 \pm \sqrt{121 + 24}}{2} = \dfrac{11 \pm \sqrt{145}}{2}$

$x = 5\tfrac{1}{2} + \tfrac{1}{2}\sqrt{145} \quad \vee \quad x = 5\tfrac{1}{2} - \tfrac{1}{2}\sqrt{145}$

e. $2\sqrt{x-1} + 8 = 15$

$2\sqrt{x-1} = 7$

$\sqrt{x-1} = 3\tfrac{1}{2}$

$x - 1 = 12\tfrac{1}{4}$

$x = 13\tfrac{1}{4}$

f. $\dfrac{4 + 2x}{x} = \dfrac{12}{x+1}$

$(4 + 2x)(x + 1) = 12x$

$4x + 4 + 2x^2 + 2x = 12x$

$2x^2 - 6x + 4 = 0$

$x^2 - 3x + 2 = 0$

$(x - 1)(x - 2) = 0$

$x = 1 \quad \vee \quad x = 2$

g. $\sqrt{2 - 2x} = -2x$

$2 - 2x = 4x^2$

$-4x^2 - 2x + 2 = 0$

$x = \dfrac{2 \pm \sqrt{4 + 32}}{-8} = \dfrac{2 \pm 6}{-8}$

$x = \dfrac{2 + 6}{-8} = -1 \quad \vee \quad x = \dfrac{2 - 6}{-8} = \tfrac{1}{2}$ (vervalt)

h. $3x = \sqrt{8x + 1}$

$9x^2 = 8x + 1$

$9x^2 - 8x - 1 = 0$

$x = \dfrac{8 \pm \sqrt{64 + 36}}{18} = \dfrac{8 \pm 10}{18}$

$x = \dfrac{8 + 10}{18} = 1 \quad \vee \quad x = \dfrac{8 - 10}{18} = -\tfrac{1}{9}$ (vervalt)

Opgave 5:

a. $\begin{cases} 3x - 2y = -5 & \vert \times 5 \\ -x + 5y = 32 & \vert \times 2 \end{cases}$

$\begin{cases} 15x - 10y = -25 \\ -2x + 10y = 64 \quad + \end{cases}$

$ 13x = 39$

$x = 3$

$9 - 2y = -5$

$-2y = -14$

$y = 7$

dus $x = 3 \quad \wedge \quad y = 7$

b. $\begin{cases} 4x + 2y = 14 & \vert \times 3 \\ 5x - 3y = 45 & \vert \times 2 \end{cases}$

$\begin{cases} 12x + 6y = 42 \\ 10x - 6y = 90 \quad + \end{cases}$

$ 22x = 132$

$x = 6$

$24 + 2y = 14$

$$2y = -10$$
$$y = -5$$
dus $x = 6 \quad \wedge \quad y = -5$

Opgave 6:
$(-4,42)$ en $(2,12)$ invullen geeft:
$$\begin{cases} -32 + 16a - 4b + 6 = 42 \\ 4 + 4a + 2b + 6 = 12 \end{cases}$$
$$\begin{cases} 16a - 4b = 68 & \big| \times 1 \\ 4a + 2b = 2 & \big| \times 2 \end{cases}$$
$$\begin{cases} 16a - 4b = 68 \\ 8a + 4b = 4 \quad + \end{cases}$$
$$\overline{\quad\quad 24a = 72}$$
$$a = 3$$
$$12 + 2b = 2$$
$$2b = -10$$
$$b = -5$$
dus $a = 3 \quad \wedge \quad b = -5$

Opgave 7:
a. $\begin{cases} a + b = 150 \\ 8{,}6a + 7b = 7{,}9 \cdot 150 \end{cases}$

$\begin{cases} a + b = 150 & \big| \times 7 \\ 8{,}6a + 7b = 1185 & \big| \times 1 \end{cases}$

$\begin{cases} 7a + 7b = 1050 \\ 8{,}6a + 7b = 1185 \quad - \end{cases}$

$\overline{\quad\quad -1{,}6a = 135}$

$a = 84{,}375$

$b = 150 - 84{,}375 = 65{,}625$

b. stel a ml van 15% oplossing en b ml van 30% oplossing

$\begin{cases} a + b = 600 \\ 0{,}15a + 0{,}3b = 0{,}22 \cdot 600 \end{cases}$

$\begin{cases} a + b = 600 & \big| \times 0{,}3 \\ 0{,}15a + 0{,}3b = 132 & \big| \times 1 \end{cases}$

$\begin{cases} 0{,}3a + 0{,}3b = 180 \\ 0{,}15a + 0{,}3b = 132 \quad - \end{cases}$

$\overline{\quad\quad 0{,}15a = 48}$

$a = 320$

$b = 600 - 320 = 280$

Dus 320 ml van de 15% oplossing en 280 ml van de 30% oplossing.

Opgave 8:

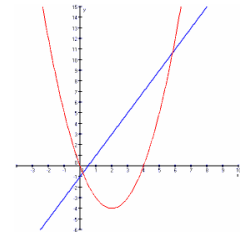a. $x^2 - 4x > 2x - 1$

$y_1 = x^2 - 4x$ en $y_2 = 2x - 1$

calc-menu intersection geeft:

$x = 0{,}17 \quad \vee \quad x = 5{,}83$
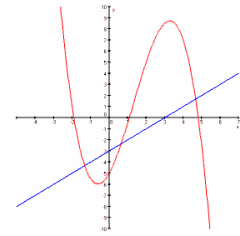
$x < 0{,}17 \quad \vee \quad x > 5{,}83$

b. $-0{,}5x^3 + 2x^2 + 3x - 5 \leq x - 3$

$y_1 = -0{,}5x^3 + 2x^2 + 3x - 5$ en $y_2 = x - 3$

calc-menu intersection geeft:

$x = -1{,}32 \quad \vee \quad x = 0{,}65 \quad \vee \quad x = 4{,}67$
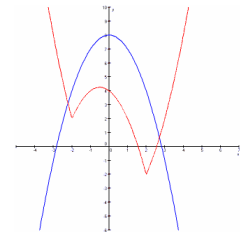
$-1{,}32 \leq x \leq 0{,}65 \quad \vee \quad x \geq 4{,}67$

c. $\left| x^2 - 4 \right| - x < 8 - x^2$

$y_1 = abs(x^2 - 4) - x$ en $y_2 = 8 - x^2$

calc-menu intersection geeft:

$x = -2{,}21 \quad \vee \quad x = 2{,}71$

$-2{,}21 < x < 2{,}71$
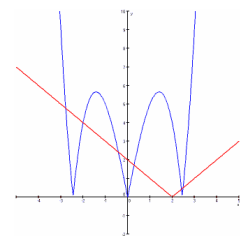
d. $\sqrt{x^2 - 4x + 4} > \left| x^3 - 6x \right|$

$y_1 = \sqrt{x^2 - 4x + 4}$ en $y_2 = abs(x^3 - 6x)$

calc-menu intersection geeft:

$x = -2{,}78 \quad \vee \quad x = -2 \quad \vee \quad x = -0{,}41 \quad \vee \quad x = 0{,}29 \quad \vee$

$x = 2{,}41 \quad \vee \quad x = 2{,}49$

$-2{,}78 < x < -2 \quad \vee \quad -0{,}41 < x < 0{,}29 \quad \vee \quad 2{,}41 < x < 2{,}49$

Opgave 9:

a. $px^3 + 2px^2 + x^2 + 2\frac{1}{4}x = 0$

$x(px^2 + 2px + x + 2\frac{1}{4}) = 0$

$x = 0 \quad \vee \quad px^2 + 2px + x + 2\frac{1}{4} = 0$ heeft twee oplossingen (dus $p \neq 0$)

$D = (2p + 1)^2 - 4 \cdot p \cdot 2\frac{1}{4} > 0$

$4p^2 + 4p + 1 - 9p > 0$

$4p^2 - 5p + 1 > 0$

$p = \dfrac{5 \pm \sqrt{25 - 16}}{8} = \dfrac{5 \pm 3}{8}$

$p = \dfrac{5 + 3}{8} = 1 \quad \vee \quad p = \dfrac{5 - 3}{8} = \frac{1}{4}$
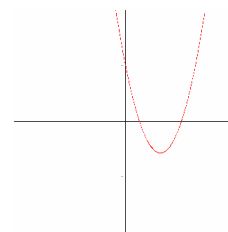
$\frac{1}{4} < p < 1$

b. $2px^4 - px^3 + 5x^3 + 2x^2 = 0$

$x^2(2px^2 - px + 5x + 2) = 0$

$x = 0 \quad \vee \quad 2px^2 - px + 5x + 2 = 0$ heeft geen oplossingen (behalve $x = 0$)

$D = (-p + 5)^2 - 4 \cdot 2p \cdot 2 < 0$

$p^2 - 10p + 25 - 16p < 0$

$p^2 - 26p + 25 < 0$

$(p-1)(p-25) < 0$

$p = 1 \quad \vee \quad p = 25$

$1 < p < 25$